MULTI-RANK METHOD: ACHIEVING PROJECTION TRANSFORMATION WITH ADJUSTABLE ACCURACY AND SPEED

Yaohua XIE [a,]*, Xiao-an TANG [a], Maoyin SUN [a], Hong CHEN [a]

[a] School of Electronic Science and Engineering, National University of Defense Technology, Changsha, P. R. China 410073 - fjpnxyh2000@yahoo.com.cn

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ABSTRACT:

In the field of GIS, it is frequently required to carry out projection transformation of massive raster maps or remote sensing images. In this paper, a method named multi-rank method is presented. It is able to achieve projection transformation with adjustable accuracy and speed. All the points in a map or image are divided into four ranks. Rank-4 points, which scatter in the map, are transformed by analytical solution. Rank-3 points are transformed by third-order polynomials, which employ rank-4 points as control points. Rank-2 points are transformed by second-order polynomials, which employ rank-3 points as control points. Rank-1 points are transformed by first-order polynomials, which employ rank-2 points as control points. Accuracy and speed of transformation can be adjusted flexibly by choosing parameters of ranking. Experimental results show that the proposed method is able to get high accuracy with high speed.

1. INTRODUCTION

In the past decades, the technologies of GIS and remote sensing developed rapidly. As a result, massive raster maps or image need to be managed and processed. For historical reasons, there are many kinds of coordinate systems of map projection, *e.g.* the Mercator projection, UTM projection and Gauss-Kruger projection. Different applications need different kinds of projection. Therefore, it's frequently required to re-project massive raster data from one system to another, *i.e.* performing projection transformations.

Presently, the most popular methods for projection transformation are analytical solution, numerical solution and analytical-numerical solution (Jiayao Wang, 2006). In analytical solution, the analytical equation between two systems must be known, and transformation is performed analytically. Numerical solution is usually employed when analytical equations are unknown; only very few common points should be known as control points, and polynomials are used to approximate analytical equations (Xiaohua Lu, 2002). Analytical-numerical solution is the simple combination of analytical solution and numerical solution; transformation between one planar system and geographical coordinate system is performed analytically, while that between the other planar system and geographical coordinate system is performed numerically. Usery *et al.* studied the extent of map projection and resampling effects on the tabulation of categorical areas by comparing the results of three datasets for seven common projections, and found significant problems in the implementation of global projection transformations in commercial software, as well as differences in areal accuracy across projections (Usery *et al.*, 2003). El-Naghi Sayed *et al*. studied transformation from Lambert Conformal projection to Transverse Mercator projection and vice versa, without referring to a spheroid (El-Naghi and Habib, 2000). Ipbuker C. proposed an iterative approach for the inverse solution of the Winkel Tripel projection using partial derivatives (Ipbuker, 2002). He and Bildirici, I. Oztug also presented an iteration algorithm to derive the inverse equations of the Winkel tripel projection, using the Newton-Raphson iteration method (Ipbuker and Bildirici, 2005). Qi Zhao *et al*. proposed an algorithm based on dual transformation; the algorithm not only gets good accuracy, but also improves transformation speed greatly (Qi *et al*., 2002). Bildirici, I. Oztug discussed two numerical methods for inverse transformation, in case the projection or projection parameters of the digitized paper map are not exactly known (Bildirici, 2003).

The above methods solve the common issues of projection transformation, but there are still some problems. Analytical solution consumes too much time because analytical equations are usually complicated. Numerical solution also consumes much time when using high-order polynomials, and gets low accuracy when using low-order ones. Presently, there are usually massive raster data in GIS systems. Therefore, it's a very important issue how to improve speed of transformation with acceptable accuracy.

In order to solve the above problems, we propose the multi-rank method which combines analytical solution and numerical solution. In the method, all the points in a map or image are divided into four ranks by several parameters. Rank-4 points, which scatter in the map, are transformed by analytical solution. Points of rank-3, rank-2 and rank-1 are transformed using polynomials of third-order, second-order and first-order respectively; these polynomials are determined by control points from rank-4, rank-3 and rank-2 respectively. The proposed method provides different solutions for different ranks. Therefore, accuracy and speed of transformation can be adjusted flexibly by several parameters.

* Corresponding author. Tel: 86-731-4576434. E-mail: fjpnxyh2000@yahoo.com.cn

2. THE PROPOSED METHOD

The essential of projection transformation is to determine the functional relationship between two planar fields, *i.e.* convert coordinates from one projection system to another. An analytical equation can be presented as follows:

$$X = F_1(x, y), \quad Y = F_2(x, y)$$

To begin with, we will provide a brief introduction on analytical solution and numerical solution, which are related to the proposed method.

2.1 Analytical Solution

Analytical solution is to perform transformation by analytical equations, which includes indirect solution and direct solution.
Indirect solution uses geographical coordinate system as a bridge. Firstly, the geographical coordinate of a point is calculated from one of its planar coordinate. Then, the other planar coordinate is calculated from the geographical coordinate. That is:

$$(x, y) \to (l, b) \to (X, Y)$$

Direct solution skips geographical coordinate; transformation is performed directly between one planar coordinate system and the other. That is:

$$(x, y) \to (X, Y)$$

2.2 Numerical Solution

Numerical solution is usually employed in case analytical equations are unknown. Firstly, certain kind of polynomial is selected as the approximation of analytical equations between two systems. Then, a group of common points, (x_i, y_i) and (X_i, Y_i), are chosen from the two systems. Using these points as control points, necessary coefficients of the polynomial are figured out. The resulting polynomial is near to the analytical equation to some extent, although not exactly the same.

The most popular polynomial in numerical solution is the nth-order polynomial. That is:

$$\begin{cases} X(x, y) = \sum_{i=0}^{n} \sum_{j=0}^{n} p_{ij} x^i y^j \\ Y(x, y) = \sum_{i=0}^{n} \sum_{j=0}^{n} q_{ij} x^i y^j \end{cases} \qquad (1)$$

Where, p_{ij} and q_{ij} is the coefficients, and $i + j \le n$.

2.3 Multi-rank Method

Analytical solution can get high accuracy, but it usually needs large calculation effort because of complicated equations. Numerical solution has high speed, especially when using low-order polynomials, but it gets greater errors than analytical solution.

The proposed method divides all the points into four ranks. Rank-4 points, which scatter in the map, are transformed by analytical solution. Rank-3 points are transformed using third-order polynomials, which are determined by control points from rank-4. Rank-2 points are transformed using second-order polynomials, which are determined by control points from rank-3. Rank-1 points are transformed using first-order polynomials, which are determined by control points from rank-2.

The detailed procedure of the method is as follows:

1. Divide the total map into blocks of $4S_4 * 4S_4$, which are called rank-4 blocks. Choose 16 points from every block as rank-4 points, and transform them by analytical solution. After that, use these points to determine the third-order polynomial corresponding to each rank-4 block.

2. Divide the total map into blocks of $3S_3 * 3S_3$, which are called rank-3 blocks. Choose 9 points from every block as rank-3 points, and transform each of them using a third-order polynomial; the polynomial is corresponding to the rank-4 block that covers the rank-3 point. If a rank-3 point is also a higher rank point, *i.e.* rank-4 point, it should not be transform again. After that, use these points to determine the second-order polynomial corresponding to each rank-3 block.

3. Divide the total map into blocks of $2S_2 * 2S_2$, which are called rank-2 blocks. Choose 4 points from every block as rank-2 points, and transform each of them using a second-order polynomial; the polynomial is corresponding to the rank-3 block that covers the rank-2 point. If a rank-2 point is also a higher rank point, *i.e.* rank-4 point or rank-3 point, it should not be transform again. After that, use these points to determine the first-order polynomial corresponding to each rank-2 block.

4. All the rest of the points are called rank-1 points. Transform each of them using a first-order polynomial; the polynomial is corresponding to the rank-2 block that covers the rank-1 point. If a rank-1 point is also a higher rank point, *i.e.* rank-4 point, rank-3 point or rank-2 point, it should not be transform again.

Choose suitable parameters that fulfill $S_4 > S_3 > S_2 > 1$. If higher rank blocks cover lower ones exactly, implementation will be easier. In that case, $4S_4 = I \cdot 3S_3$ and $3S_3 = J \cdot 2S_2$ where $I, J \in N$.

3. EXPERIMENT AND ANALYSIS

In order to test the accuracy and speed of the proposed method, we implement it using Matlab7.0. As the task is to transform all the points from one coordinate system to another, no actual coordinates are needed; what must be done is to step through the total map. Our concern is mainly on massive raster image, therefore, it was assumed that the global image is $G \times G$ in pixel, where G equals 4000000; that is about 10 meters per pixel. It takes too much time to transform so many points. Therefore, we chose five representative areas, which are shown in Figure 1. Where, the white square represents the global image. The gray small squares represent the chosen area. Each square is of size $R \times R$, where R equals 1152.

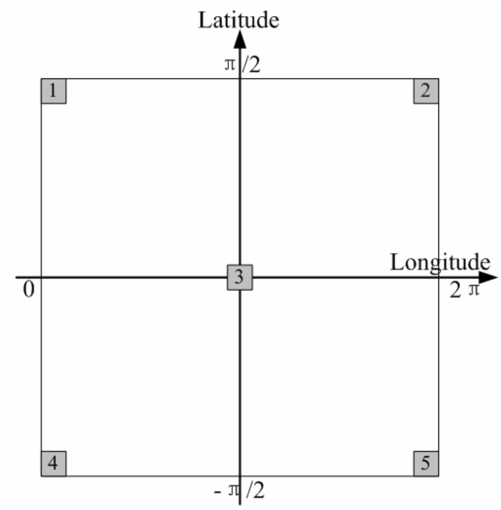

Figure 1. Locations of five representative areas.

Transformation is performed from planar coordinate system to UTM coordinate system. Firstly, the analytical equation from planar coordinate (u,v) to geographical coordinate (l,b) is:

$$\begin{cases} l = \dfrac{2\pi \cdot u}{G-1} \\ b = \pi \cdot \left(\dfrac{1}{2} - \dfrac{v}{G-1} \right) \end{cases} \tag{2}$$

Given that the origin latitude is 0 and the origin longitude is l_0. So that the analytical equation from geographical coordinate (l,b) to UTM coordinate (X,Y) is:

$$\begin{cases} X = FE + k_0 N \cdot [A + (1-T+C) \cdot \dfrac{A^3}{6} \\ \qquad + (5 - 18T + T^2 + 72C - 58e'^2) \cdot \dfrac{A^5}{120}] \\ Y = FN + \\ k_0 \cdot \{ M + N \cdot tgB[\dfrac{A^2}{2} + (5 - T + 9C + 4C^2) \cdot \dfrac{A^4}{24}] \\ \qquad + (61 - 58T + T^2 + 600C - 330e'^2) \cdot \dfrac{A^6}{720} \} \end{cases} \tag{3}$$

Where, FE equals 500000, in meter; FN is 0 in the Northern Hemisphere and 10000000 in the Southern Hemisphere, in meter; k_0 equals 0.9996. The rest are:

$$\begin{cases} T = tg^2 b \\ C = e'^2 \cos^2 b \\ A = (l - l_0) \cos b \\ M = \alpha \cdot [(1 - \dfrac{e^2}{4} - \dfrac{3e^4}{64} - \dfrac{5e^6}{256}) b - (\dfrac{3e^2}{8} + \dfrac{3e^4}{32} + \dfrac{45e^6}{1024}) \sin 2b \\ \qquad + (\dfrac{15e^4}{256} + \dfrac{45e^6}{1024}) \sin 4b - \dfrac{35e^6}{3072} \sin 6b] \\ N = \dfrac{\alpha}{\sqrt{1 - e^2 \cdot \sin^2 b}} = \dfrac{(\alpha^2/\beta)}{\sqrt{1 - e'^2 \cdot \cos^2 b}} \end{cases} \tag{4}$$

Where, α, β, e, e' are the parameters of Earth ellipsoid. That is:

$$\begin{cases} \alpha : semi-major\ axis \\ \beta : semi-minor\ axis \\ e = \sqrt{1 - (\beta/\alpha)^2} \\ e' = \sqrt{(\alpha/\beta)^2 - 1} \end{cases} \tag{5}$$

We chose the parameters as: $S_4 = 36 \cdot k$, $S_3 = 12 \cdot k$, $S_2 = 6 \cdot k$; $k = 1,2,4,8$, so that the performance of the proposed method can be tested under four different configurations. Points of all ranks are chosen by the following formulas, so that they scatter in their area and are not too regular.

$$\begin{cases} u_4 = \tau_4 + \dfrac{S_4}{4} \cdot m_4 + S_4 \cdot n_4 \\ v_4 = \varphi_4 + S_4 \cdot m_4 + \dfrac{S_4}{4} \cdot n_4 \\ \tau_4, \varphi_4 = 0, 4S_4, \cdots (M - 4S_4); \\ m_4, n_4 = 0, 1, 2, 3; \end{cases} \tag{6}$$

$$\begin{cases} u_3 = \tau_3 + \dfrac{S_3}{3} \cdot m_3 + S_3 \cdot n_3 \\ v_3 = \varphi_3 + S_3 \cdot m_3 + \dfrac{S_3}{3} \cdot n_3 \\ \tau_3, \varphi_3 = 0, 3S_3, \cdots (M - 3S_3); \\ m_3, n_3 = 0, 1, 2; \end{cases} \tag{7}$$

$$\begin{cases} u_2 = \tau_2 + \dfrac{S_2}{2} \cdot m_2 + S_2 \cdot n_2 \\ v_2 = \varphi_2 + S_2 \cdot m_2 + \dfrac{S_2}{2} \cdot n_2 \\ \tau_2, \varphi_2 = 0, 2S_2, \cdots (M - 2S_2); \\ m_2, n_2 = 0, 1; \end{cases} \tag{8}$$

Where, (u_4, v_4), (u_3, v_3) and (u_2, v_2) are the coordinates of points of rank-4, rank-3 and rank-2, respectively.

Analytical solution and numerical solution (using first-order polynomial) are taken as comparison. The former transforms all the points by analytical equations. The latter chooses the four corner points of each area as control points; after a first-order polynomial is determined, all the other points in the image are transformed by it. Experimental results of area 1 to 5 are shown in Table 1-5, where time consumption (denoted as TC, in second) is of all the points in an area. The definition of error is:

$$e = \frac{1}{R^2} \cdot \sum_{i=0}^{R-1} \sum_{j=0}^{R-1} \left| f(i,j) - g(i,j) \right| \quad (9)$$

Where, $f(i,j)$ is the result of analytical solution, $g(i,j)$ is the result of multi-rank method or numerical solution.

Table 1. Result of area 1

k	**TC of analytical solution**	**TC of multi-rank method**	**TC of numerical solution**	**error of multi-rank method**		**error of numerical method**	
				u	***v***	***u***	***v***
1	87.5156	22.0469	16.9219	1.18×10^{-6}	9.56×10^{-5}	0.0011	0.6546
2	87.5000	20.0000	16.7031	2.52×10^{-6}	3.95×10^{-4}	0.0011	0.6546
4	87.6406	19.4063	16.7969	4.07×10^{-6}	0.0016	0.0011	0.6546
8	87.5469	19.2656	16.8125	1.53×10^{-5}	0.0064	0.0011	0.6546

Table 2. Result of area 2

k	**TC of analytical solution**	**TC of multi-rank method**	**TC of numerical solution**	**error of multi-rank method**		**error of numerical method**	
				u	***v***	***u***	***v***
1	88.0781	21.9844	16.7188	0.1982	0.7334	22.9536	30.3602
2	88.4531	20.0938	16.7969	0.2011	0.7345	22.9536	30.3602
4	89.0000	19.4063	16.8281	0.2190	0.7451	22.9536	30.3602
8	88.4219	19.2500	16.7813	0.3271	0.8284	22.9536	30.3602

Table 3. Result of area 3

k	**TC of analytical solution**	**TC of multi-rank method**	**TC of numerical solution**	**error of multi-rank method**		**error of numerical method**	
				u	***v***	***u***	***v***
1	88.9375	22.0156	16.7500	0.0207	0.0050	19.1116	44.8892
2	89.1719	20.0469	16.7500	0.0257	0.0188	19.1116	44.8892
4	88.9219	19.4375	16.7813	0.0538	0.0763	19.1116	44.8892
8	88.8906	19.2813	16.7344	0.1890	0.3103	19.1116	44.8892

Table 4. Result of area 4

k	**TC of analytical solution**	**TC of multi-rank method**	**TC of numerical solution**	**error of multi-rank method**		**error of numerical method**	
				u	***v***	***u***	***v***
1	89.0469	22.1094	16.8594	1.25×10^{-6}	9.54×10^{-5}	0.0011	0.6546
2	89.3906	19.9844	16.8281	2.17×10^{-6}	3.95×10^{-4}	0.0011	0.6546
4	89.4063	19.3906	16.9063	3.99×10^{-6}	0.0016	0.0011	0.6546
8	88.1563	19.2969	16.7813	1.53×10^{-5}	0.0064	0.0011	0.6546

Table 5. Result of area 5

k	**TC of analytical solution**	**TC of multi-rank method**	**TC of numerical solution**	**error of multi-rank method**		**error of numerical method**	
				u	***v***	***u***	***v***
1	90.2813	21.9844	16.8594	0.1982	0.7335	22.9536	30.3602
2	90.0000	20.0469	16.7188	0.2011	0.7345	22.9536	30.3602
4	90.2813	19.5000	16.8750	0.2191	0.7453	22.9536	30.3602
8	89.8438	19.2500	16.8125	0.3277	0.8299	22.9536	30.3602

As shown by the results, the errors of the proposed method are far less than those of numerical solution; its time consumption is also much less than that of analytical solution, and them are near to those of numerical solution; furthermore, the greater the value of k, the less the time consumption. By choosing k (or S_4, S_3, S_2) properly, accuracy and speed of transformation can be adjusted flexibly, so as to meet the needs of various applications. In fact, analytical solution and numerical solution can be treated as special cases of multi-rank method.

4. CONCLUSION AND DISCUSSION

In this paper, a method, named multi-rank method, is presented for projection transformation of massive maps or images. The method has adjustable accuracy and speed of processing. It combines existing analytical solution and numerical solution. Points in a map are divided into four ranks, and different approaches are adopted for them. By choosing suitable parameters, accuracy and speed can be adjusted flexibly. Experimental results show that the proposed method can achieve rather high accuracy with little time consumption. The method can be applied in many fields such as GIS, remote sensing, *etc.*

The distribution of control points affects the accuracy of polynomial coefficients to some extent. What was adopted in this study is not necessarily the optimal distribution. In the future, some work should be done to study how accuracy changes with the distribution of control points, and then find the optimal strategy to choose control points, so as to further improve accuracy.

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