An optimal interpolation scheme for producing a DEM from the automated stereo-matching of full-scale SPOT images

Seungbum Kim*, Taejung Kim*, Wonkyu Park*, Heung-Kyu Lee**
* Satellite Technology Research Center, ** Dept. of Computer Science, Korea Advanced Institute of Science and Technology, 373-1 Kusung, Yusung, Taejeon, S. Korea 305-701 E-mail: sbkim{tjkim}@krsc.kaist.ac.kr

Keyword: Optimal interpolation, Digital Elevation Model, SPOT, Linked list, Decorrelation scale, Multiquadric, Modified Shepard, Minimum Curvature, Kriging.

Abstract: Eight interpolation methods are implemented and tested over a 60 km × 60 km region in Korea, using the results from automatic stereo-matching of SPOT images. First, with the simulated elevation data from digitised contours of paper maps, Kriging and Multiquadric methods show higher performance: with the accuracy better than 3 m than other methods and the speed of ~400 s on a Sparc 20 machine. Second, in a realistic case when the elevation derived from SPOT images is used, Kriging and Gaussian perform better than other methods by 1 to 5 m in terms of accuracy. Due to the errors in the SPOT stereo-match output, Gaussian appears to perform better than Kriging. As a third experiment, we consider large regions of match-failure due to cloud cover, image skewness, and grey-value differences. To prevent interpolation on these regions, a 'centre-of-gravity' within each search region of interpolation is used as a rejection criterion.

1. Introduction
A digital elevation model (DEM) is one of the most widely used medium for terrain analysis and constructing geographical information system. DEMs from satellite photos have gained increasing advantages over aerial DEMs thanks to regular scanning, unrestricted access to the global terrain, and high-resolution (up to 1-m) missions. DEM generation involves the following three procedures: stereo matching, sensor modelling, and DEM interpolation. Stereo-matching produces conjugate points from a stereo image pair. These points are in an image coordinate and converted into a ground coordinate by sensor modelling. After sensor modelling, the elevation values do not provide a complete spatial coverage. Such an incomplete coverage would occur due to cloud cover, image skewness, grey level difference, and limitations of a matching algorithm. A complete coverage may be obtained by interpolating scattered elevation values. Also, if stereo-matching is performed at a coarse resolution for computational reasons (e.g., at every 50 m), interpolation is used to generate a DEM with higher density. In this way interpolation becomes a crucial element determining coverage and accuracy of a DEM.

Schemes for interpolating elevation data have been studied (e.g., Renka 1988; Carlson and Foley 1991; Desmet 1997 and, for survey papers, see Schumaker 1976 and Franke 1982a). Most of these studies do not deal with real elevation data but uses a test data set with less than 100 elements. In contrast, SPOT stereo-match results generate a significant number of data (O(6)), are unevenly scattered, and contain large areas of match failure. In this paper, for the first time, the quality assessment of various interpolation methods is performed based on SPOT stereo-match results.

Sections 2 and 3 describe data and the interpolation methods. In sections 4 and 5, the performance of each interpolation method is presented when the simulated data and the real stereo-match results over a small area are used, respectively. In section 6, interpolation over the SPOT full-scene area is discussed.

2. Data
The DEM is generated using a SPOT stereo-pair, taken 11:23am on Oct. 20, 1997 and 11:12am on May 22, 1998. The tilt angles are 13.2º to the east and 10.2º to the west, with the base-to-height ratio of about 0.4. The area covered is of about 60 km by 60 km. Stereo-matching is an area-based one using the epipolar correlation criterion (Kim submitted). About 20 GPS measurements of latitude, longitude and elevation are used as ground control points (GCP). To save computing time, the stereo-matching is performed at every 5 pixel, thus the output resolution is 50 m. 528,254 points are matched out of 1,440,000 grid (Fig. 1). The coverage is somewhat low because in the southern two-fifth of the region, there no GCPs. In the Northwestern and Southeastern parts of the region, the terrain is flat at elevations less than 200 m. In the Northeastern and Southwestern parts, there are mountains with elevation ranging from 200 m to 1000 m, which occupy about 40% of the entire region.
A digitised DEM is used to provide scattered elevation. The DEM is obtained by digitising 1:50,000 map of the Korean National Geographic Institute. Digitisation is performed using the ScanGraphics drum scanner with 500 dpi resolution and using a 44 inch-wide map. After interpolation, the resulting map is provided at 60-m resolution and is of size, 749 × 464 pixels.

3. Methodology
3.1. Interpolation method
Eight interpolation methods are implemented. The parameters for each method are determined so that they perform best with the input scattered data.

Bin Average (exact interpolation)
Bin Average is the simplest of all methods. The input data are stored in a structure called “bin”, a rectangle centred on an output grid.

Moving Window Average (MWA, smooth interpolation)
Moving Window Average assigns to the output the average of the input elevations within an interpolation radius.

Nearest Neighbourhood (NN, exact interpolation)
The output is the height of the input point whose distance to the grid location is the shortest among input points.

Kriging (exact interpolation)
Kriging aims at finding the optimal weights \( \omega_i \) for interpolation of the height \( H \) at \( x_o \), by utilising the statistical properties of a terrain:

\[
\hat{H}(x_o) = \sum_{i=1}^{n} \omega_i H(x_i).
\]

The optimal weights are determined using a function that describes the spatial variation of a terrain (called variogram). The variogram is defined, subjectively in this paper, as

\[
1.5 x/2R - 0.5 x/2R^2,
\]

with \( R \) being the Kriging radius and \( r \) the distance from a grid cell. Ideally a variogram should be determined by analysing the terrain of interest, but this requires significant amount of time. Full details of Kriging can be found in Wakernagel (1995).

Gaussian (smooth interpolation)
Gaussian interpolation uses Eq. 1 but the weighting function is given by \( \exp(-\frac{(x - x_o)^2}{\sigma^2}) \). Here \( \sigma \) defines the width of the Gaussian function and is set to one third of the interpolation radius.

Modified Shepard (exact interpolation)
This method (Renka 1988) uses quadratic basis functions and inverse-distance weights. The details that Renka used are found appropriate in our data set, thus are adopted.

Multiquadric (exact interpolation)
Multiquadric method uses the same form as Eq. 1 except that the basis function is given as:

\[
Q_i(x, y) = \sqrt{(x - x_i)^2 + (y - y_i)^2 + R^2}.
\]

The coefficient \( R \) is regarded crucial (Carlson and Foley 1991), and

\[
R^2 = \left[1.25D/\sqrt{n}\right] \left[125D/\sqrt{n}\right] \left[125D/\sqrt{n}\right]
\]

is chosen empirically using the digitised map of § 2 (Table 1). Minimum curvature (exact interpolation)
Minimum Curvature interpolation aims at minimising the curvature of the surface that are defined by interpolated elevation. The interpolant formula presented in Franke (1982b) is adopted.

Data structure
Careful choice of data structure can save computing time. A two dimensional bin array is constructed, and input scattered records are allocated to corresponding bins. There can be more than one scattered points per bin. To handle this, a linked list is employed. Interpolation algorithm is first to search bins within an interpolation radius, then to include all the scattered points in the linked list.

Table 1. Determination of the user-defined constant \( R^2 \) using the digitised DEM in § 2. Statistics are RMS of an interpolated DEM with respect to the digitised DEM. \( D \) is the horizontal boundary of input scattered records and \( n \) their number. \( n \) varies in each search area.

<table>
<thead>
<tr>
<th>Sampling density</th>
<th>( R^2 )</th>
<th>( \left[0.125D/\sqrt{n}\right] )</th>
<th>( \left[1.25D/\sqrt{n}\right] )</th>
<th>( \left[125D/\sqrt{n}\right] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1%</td>
<td>46.02</td>
<td>43.90</td>
<td>3835.56</td>
<td></td>
</tr>
<tr>
<td>45%</td>
<td>11.94</td>
<td>5.34</td>
<td>16.22</td>
<td></td>
</tr>
<tr>
<td>99.7%</td>
<td>1.16</td>
<td>0.28</td>
<td>1.44</td>
<td></td>
</tr>
</tbody>
</table>
3.2 Determination of the maximum number of scattered points

Use of less points will increase the speed of interpolation (however, at the cost of reducing its accuracy). Instead of including all the points within the search radius, interpolation is performed using the nearest \( N \) points within the radius. \( N \) is determined empirically using sample points from the digitised DEM of § 2. As a result of the compromise between speed and accuracy \( N \) is set to 10 (Table 2).

Table 2. Determination of the maximum number of points to included during interpolation. The scattered data come from the digitised map \((749 \times 464)\). The speed is measured on a Sparc 20 workstation. The RMSE is computed by comparing the interpolation result with the digitised map.

<table>
<thead>
<tr>
<th># of scattered points</th>
<th>Speed (sec)</th>
<th>Accuracy (RMSE, m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
<td>6000</td>
<td>19.33</td>
</tr>
<tr>
<td>20</td>
<td>2072</td>
<td>19.35</td>
</tr>
<tr>
<td>15</td>
<td>1041</td>
<td>19.43</td>
</tr>
<tr>
<td>10</td>
<td>414</td>
<td>19.69</td>
</tr>
<tr>
<td>5</td>
<td>111</td>
<td>20.79</td>
</tr>
</tbody>
</table>

3.3. Determination of the interpolation radius

There are several ways to determine the interpolation radius \( R \). First, an analysis of a variogram can give \( R \). In practice, however, the shape of a variogram is often far from an ideal one. One may use sample data such as the digitised DEM in § 2, and empirically determine \( R \) so that the interpolation error may be minimised. In this work, first, the minimum density of scattered points, that interpolation should be able to handle, is set to 1%. Then, if the optimum number of scattered points (10 in § 3.2) are to be included, \( R \) should be 9 pixels.

This criterion sets the minimum value of \( R \) (Fig. 2). In Fig. 2, the concept of decorrelation scale is then used to fix \( R \). The autocorrelation map shows, for the correlation > 0.3, that the maximum correlation occurs in the NW-SE direction. This is due to the presence of the flat area in the direction. The rough terrain in NE-SW direction makes the correlation in the direction low. In the case of smooth topography such as the ocean, a decorrelation scale is where autocorrelation becomes zero (Qiu et al. 1991). Roughness of land surface makes it difficult to adopt this concept directly. Here the interpolation radius is set to 30 pixels or 1.8 km (at 60 m resolution), where the autocorrelation is about 0.7. This interpolation radius is used by all the interpolation methods.

4. Analysing the performance of interpolation: using simulated input

The first experiment for assessing the quality of interpolation methods is to use simulated data set. There are two objectives here: first, to determine the optimum interpolation method; second, to examine what is the minimum number of scattered points to produce an interpolated DEM of acceptable quality.

The simulated data set is prepared by random-sampling of the digitised DEM (see § 2). Interpolation outputs are compared with the original DEM to assess the accuracy of interpolation. In order to examine the effect of the number of sample points, interpolation is repeated with 21 different sampling density ranging from 1% to 99.7%.

Visual comparison

Fig. 3 shows the digitised DEM and interpolated DEMs using NN, Gaussian, and Kriging. Kriging interpolation produces a smooth and visually reasonable DEM even with this scanty data (so does Minimum Curvature and Multiquadric). NN introduces block artifacts. Gaussian result is very smooth.

What is the minimum level of sampling density, to guarantee an interpolated DEM of acceptable quality? When the sampling density is above 45%, changes caused by increasing sampling density are indiscernible by visual comparison (not shown). Quantitative support is provided below.

Accuracy vs. interpolation method

Fig. 4 shows the accuracy of each interpolation method versus the sampling density. As expected, root mean square error decreases as the number of input points increases. It is observed that:

- in all cases, Minimum Curvature produces the
Fig. 3. DEMs created using 1% sampling density data by (from the top-left, clockwise) Nearest Neighbour, Kriging, reference DEM, and Gaussian. Bright colour corresponds to large elevation.

most accurate DEM. It is more accurate than Kriging by on average 0.7 m [user11].

- Multiquadric method has lower accuracy than Kriging by on average 0.7 m (as in Lee (1996)).
- Smooth interpolation methods (MWA and Modified Shepard) perform very poorly. A part of the poor performance may be due to the polynomial regression scheme which is included in modified Shepard method. In Lee (1996), the polynomial regression produces the least accurate DEM. However, the accuracy improves fast with sampling density, notably at the density > 80%. This is thanks to the exact interpolation.

- 30 to 45% sampling density is the nodal point in error reduction: below this level, the error varies sharply (from ~ 80 down to 10 m); above this level, the variation is slow (from ~ 10m to 1 m, or even slower for the smooth interpolation). This agrees with the visual comparison result that 45% sampling density produces almost identical DEM to the reference DEM.

Speed vs. interpolation method

Fig. 5 shows comparisons of processing time. Minimum Curvature requires the longest processing time. By comparison Bin Average, MWA, NN, and Gaussian runs faster by about 100 times. Kriging, Multiquadric, and Modified Shephard processes at medium speed, at about 1/10th of that of Minimum Curvature. Further analyses show that:

- Bin Average is the fastest method because time required to input points into 2D bin array is the dominant factor. NN runs faster than the Bin Average near 100% sampling density. This is because at these densities searching nearest
neighbour points is quicker than searching scattered points within a bin.

- Gaussian, MWA, and NN require the smaller amount of time since they do not include complex mathematical operations, such as solving a linear set of equations to define the basis functions.

- Multiquadric interpolation works at a similar speed to that of Kriging: Multiquadric is slightly faster by on average 73 seconds than Kriging.

- Modified Shepard method consumes twice as much time as Kriging to process the 1% density input. The processing time drops sharply with sampling density, for example, by 20 times at 94% density. Such marked reduction is achieved by skipping interpolation routine completely when an input elevation is available at an output grid.

- it is noticeable that Minimum Curvature is slow by more than 10 folds. This extremely slow performance is the consequence of using Singular Value Decomposition (SVD) routine, which is used to define the basis function. It takes 1473 seconds to solve a linear equation of order 13 using SVD. For an equation of order 8, the processing time is reduced to 471 seconds.

- a smaller number of input points requires more time (except for Bin Average). The reason is that it takes more time to search the nearest \( N \) points at low sampling density.

- processing time for Gaussian, MWA, Kriging, Multiquadric, and Minimum Curvature is by and large independent of the number of input points at sampling density \( > \sim 30 \% \). This is because data search is no longer a main factor of time consumption at these densities. This finding suggests that the interpolation method implemented here, specially its data structure, is capable of dealing with input scattered points at the speed independent of the data size.

---

**5. Optimal interpolation: using an evenly-spaced stereo-match results**

In this section, real stereo-match results rather than the simulated data in § 4 are used to find an optimal interpolation method. The purpose is to examine whether the effective methods in § 4 show superior performance here too. The distribution of the simulated data in § 4 is more or less even thanks to random sampling. Thus terrains with similar properties are selected: two small boxes of about 0.1° × 0.1° size in Fig. 1. The two regions have 90% match-coverage, with one being smooth terrain and the other rough. Five interpolation methods are tested: Gaussian representing smooth interpolation...
and four exact interpolation methods (Minimum Curvature, Kriging, Modified Shepard, and Multiquadric).

Kriging and Gaussian perform best (Table 3). The accuracy of Kriging is 37.8 and 44.4 m for the two regions, respectively, when compared with the Digital Terrain Elevation Model. With respect to Kriging accuracy, the accuracies of Gaussian are within +/- 0.5 m, and those of other methods increase by 1 to 5 m [sbkim14]. The superiority of the smooth interpolation, Gaussian, is in contrast to the result with the simulated data: smooth interpolation performs poorly (Fig. 4). This contrast arises from the errors in the stereo-match results. These errors are represented in Fig. 6a by the spikes (since Kriging is the exact interpolation, errors in the stereo-match results appear unfiltered in the interpolation output). With Gaussian, these errors are smoothed out1(Fig. 6b).

Table 3. Quality assessment of interpolation, performed using the SPOT stereo-match results over the two 0.1º × 0.1º size in Fig. 1. The truth DEM is the US NIMA (National Imagery Mapping Agency)’s DTED (Digital Elevation Terrain Model) level 1 (100 m horizontal resolution).

<table>
<thead>
<tr>
<th>Interpolation method</th>
<th>Accuracy (RMSE, m)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Area 1</td>
</tr>
<tr>
<td>Gaussian</td>
<td>36.55</td>
</tr>
<tr>
<td>Minimum Curvature</td>
<td>39.32</td>
</tr>
<tr>
<td>Kriging</td>
<td>37.84</td>
</tr>
<tr>
<td>Multiquadric</td>
<td>40.54</td>
</tr>
<tr>
<td>Modified Shepard</td>
<td>39.04</td>
</tr>
</tbody>
</table>

Gaussian appears to give not only comparable but superior result to Kriging (Fig. 6). If other topographic variables such as slope are used for quality assessment, a quantitative result may be able to support this.


In Fig. 1, there are large regions where stereo-match failed. Interpolation over these regions introduces unrealistic elevation, and this can be explained using the concept of ‘centre-of-gravity (COG)’ and ‘empty-centre-index (ECI)’ (Fig. 7). Without correcting for the COG, boundaries of matched regions would have incorrect elevation (Fig. 8). Even if the COG criterion is satisfied, if stereo-match fails over a small hill, the hill would not be correctly represented after interpolation. Thus the ECI criterion is also needed. The threshold values of the COG and ECI are both set to 0.4.

Fig[skkim16]. 7. Concepts of ‘centre-of-gravity’ (COG left) and ‘empty-centre-index’ (ECI, right). The ECI can be regarded as a 1D version of the COG in the radial direction. The shaded area is where stereo-match succeeded, while the white area is not. The outer circle indicates an interpolation radius.

Fig[skkim17]. 8. The effect of applying the centre-of-gravity (COG) criterion: when the COG is not applied (left) and applied (right). The area is of about 10 km × 10 km. Gaussian interpolation is applied with the interpolation radius of 30 pixels.

Fig. 9 shows the impact of using the COG and ECI criteria over the SPOT full-scene region. In comparison to the locations of stereo-match results before interpolation (Fig. 1), interpolation completes the small holes (noticeable in the northwestern part of the area). Without the COG and ECI criteria, the clouded region (the long stretch with the NW-SE direction in the western part of the image) and the large holes in the northeast are filled (Fig. 9a). Interpolation in these empty regions would be acceptable for a hill side, but not so for the tops of a mountain or a hill. Along the southern boundary of the image, the circular-striped artifacts in Fig. 9a look unrealistic. In Fig. 9b, such unwanted interpolation are prevented.

1 Precisely speaking, smooth interpolation spreads an error at one point to its neighbour, not eliminating it. The spreading, however, reduces the RMSE: RMS of a smoothed surface is smaller than that of the original surface.
7. Conclusions

Eight methods are implemented to interpolate stereo-match results for full-scale SPOT images. They are Bin Average, Nearest Neighbour, Moving Window Average, Gaussian, Kriging, Multiquadric, Modified Shepard, and Minimum Curvature. Their characteristics and performances are analysed and the most effective method is searched.

To compare the performances of the eight methods, the simulated data are generated by random sampling of a digitised map over 30 km × 40 km region in Korea (749 rows × 464 columns). The sampling is made at 21 density levels from 1% to 99%. The quality of an interpolation method is assessed in terms of the accuracy of the interpolated DEM. The accuracy is computed by comparing with the original DEM. The analysis show:

- although Minimum Curvature is the most accurate, it is at least 10 times slower. Considering both speed and accuracy, Kriging and Multiquadric interpolation perform best.
- 45% is the minimum density required, of scattered points (Fig. 4): above this level, the interpolated DEMs output show little visual difference from the truth and the quantitative error varies only very slowly (by less than 10 m at maximum).
- for the density of scattered points > about 45 %, the processing time is independent of the size of input data (Fig. 5). Next, the real data from SPOT stereo-match results are interpolated, rather than the simulated data. Initially two 10 km × 10 km regions are chosen with almost evenly-scattered points. Kriging and Gaussian perform best. The accuracy of Kriging is 37.8 and 44.4 m for the two regions, respectively, when compared with the Digital Terrain Elevation Model. With respect to Kriging accuracy, the accuracies of Gaussian are within ±0.5 m, and those of other methods increase by 1 to 5 m[skkim18]. The smooth interpolation shows good performance both visually (Fig. 6) and quantitatively. The reason is that the elevation errors in the stereo-match results are smoothed.

At a full-scale (60 km × 60 km), stereo-match results contain large regions of match failure. Interpolation over these areas often result in large errors. By using the centre-of-gravity and the empty-centre-index, interpolation over such as empty area is prevented.

Acknowledgement

The work presented here is supported by the Ministry of Science and Technology of the Government of Korea.

References


English abstract).


