BLOCK ADJUSTMENT

Karsten Jacobsen

Institute for Photogrammetry and Surveying Engineering
University of Hannover
FAX +49 511 762 2485
Email karsten@ipi.uni-hannover.de
http://www.ipi.uni-hannover.de

1. INTRODUCTION

The aerial triangulation or block adjustment today is a basic tool in the photogrammetric data handling. For each photogrammetric purpose the photo orientations must be know. It is not economic to determine the photo orientation individually for each photo or model based on control points. Up to now the direct determination of the exterior orientation – called direct sensor orientation - has a limited accuracy, which is sufficient only for few cases. The determination of the projection centers by GPS has been improved from year to year and is used in the combined block adjustments.

2. DEVELOPMENT OF THE BLOCK ADJUSTMENT

The determination of control points for the orientation of each individual photogrammetric model is expensive and time consuming, so together with the development of the analog photogrammetric instruments also the connection of successive photos was introduced as well as the slotted template method for the reduction of the number of required control points. The slotted template method is not in use today anymore. The strip adjustment can be based on photo or model coordinates. Neighbored models can be connected by similarity transformation using also the model coordinates of the projection centers. The main problem of the photo strips are the systematic deformations caused by the summation of systematic and random errors.

Caused by the development and availability of modern computers the unfavorable error propagation of analog photo strips was improved by simple strip adjustments, followed by anblock adjustment and the block adjustment by independent models. The anblock adjustment is based on leveled models and is a common similarity transformation of the models to the control points. This method is not used anymore because it is limited to the horizontal components. The block adjustment by independent models was the first really three-dimensional method. As observations model coordinates are used corresponding to the method of data acquisition in the 1960th. Of course the model coordinates can be computed also today based on photo coordinates, but the loss of information by the relative orientation and the changing combination of systematic image errors to systematic model errors is degrading the accuracy against the most rigorous method, the bundle block adjustment. The latter method also includes beside of the higher accuracy some advantages for the combined block adjustment with projection center coordinates determined by relative kinematic GPS-positioning. Also for the determination of the boresight misalignment for the direct sensor orientation a bundle block adjustment is required as reference because the 6 parameters of the exterior orientation have to be known.

3. METHODS OF BLOCK ADJUSTMENT

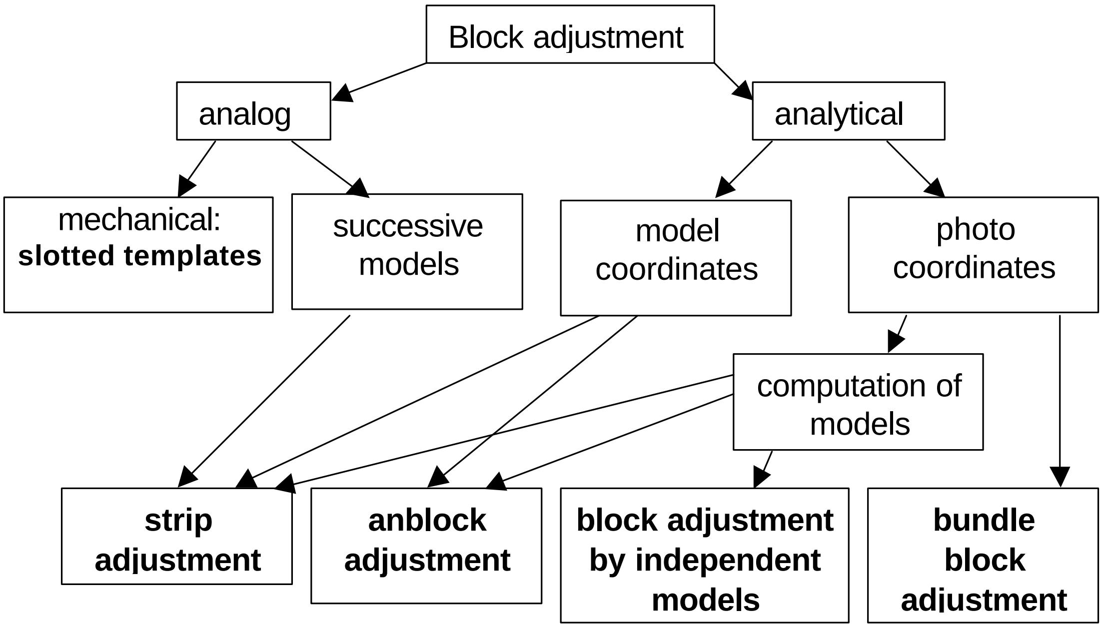


figure 1: methods of block adjustment

3.1 STRIP ADJUSTMENT

A strip coordinate system can be created by the connection of successive photos in analog stereo plotters or by the calculation of model coordinates based on photo coordinates and the transformation of neighbored models together. This internal strip coordinate system can be transformed to the control points by similarity transformation. The major disadvantage of this method is the deformation of the strips caused by a summation of systematic and random errors.

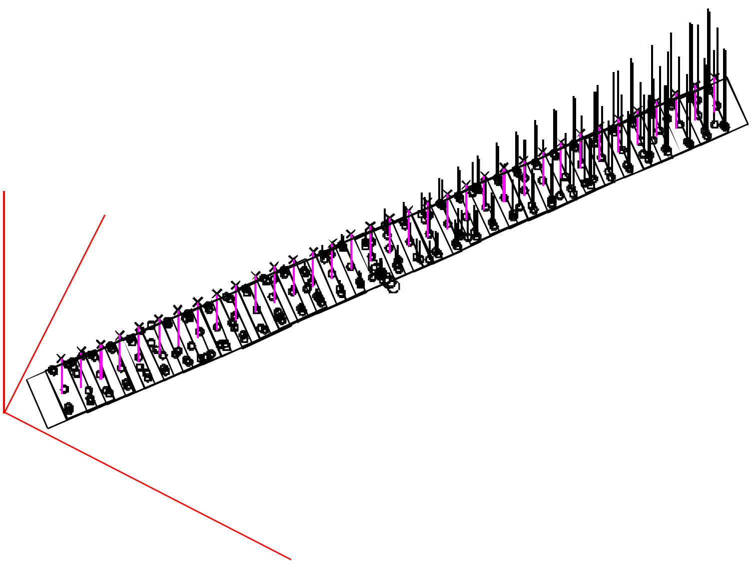


figure 2: vertical deformation of a photo strip

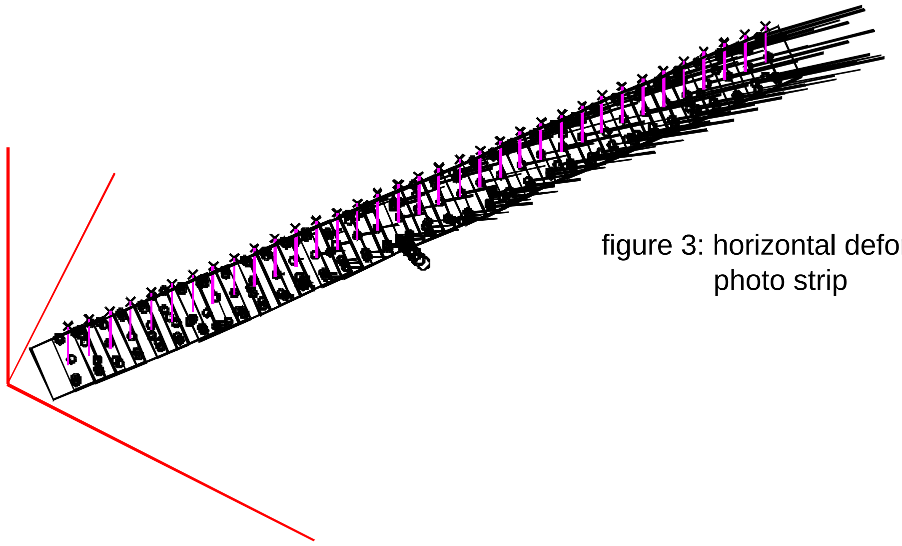


figure 3: horizontal deformation of a photo strip

As it can be seen in figure 2 and 3, photo strips connected to control points only at one end of the strip are strongly deformed. The deformation can be reduced in using a polynomial adjustment based on control points.

$$DX = A + B \cdot X + C \cdot X^2 \quad + D \cdot X \cdot Y + E \cdot Y \quad + F \cdot X^3 + G \cdot X^2 \cdot Y$$

formula 1: polynomials used for strip adjustment

The polynomial adjustment will be done separately for all 3 components. The degree is depending upon the number and distribution of control points, 3, 5 or 7 unknowns are used. In general this is only an approximate computation and not a rigorous adjustment, so the final accuracy is limited and the method should be used only as an a priori computation.

3.2 BLOCK ADJUSTMENT BY INDEPENDENT MODELS

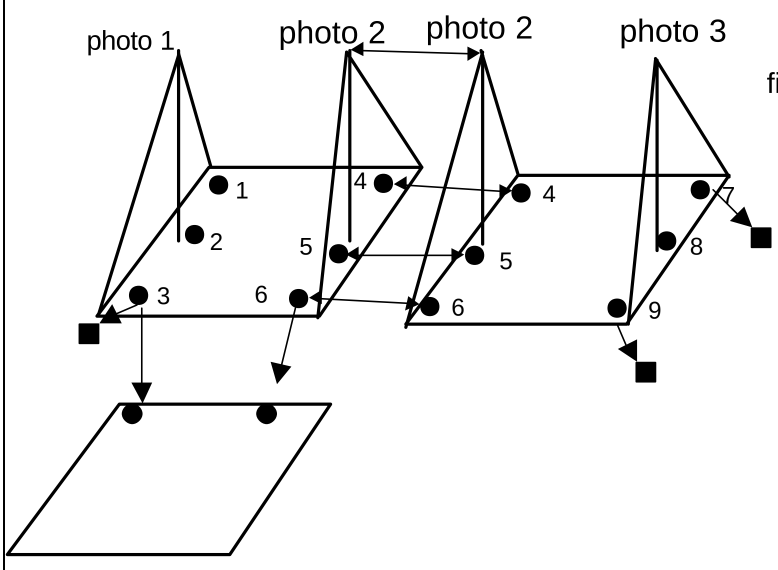


figure 4: block adjustment by independent models

The block adjustment by independent models is based on model coordinates including also the model coordinates of the projection centers. The mathematical model is a simultaneous similarity transformation of the models fixed by tie points to the control points. By theory 2 horizontal and 3 vertical control points are required, but for a sufficient accuracy more control points are necessary.
The method is still in use today, even if the data acquisition is based on photo coordinates. But the loss of information caused by the relative orientation (4 photo coordinates → 3 model coordinates) and the corresponding loss of accuracy, especially in the height, should not be accepted, it is not anymore the state of the art.

3.3 BUNDLE BLOCK ADJUSTMENT

The mathematical model of the bundle block adjustment is the perspective attribute of the metric photos, as formula the collinearity equation is used.

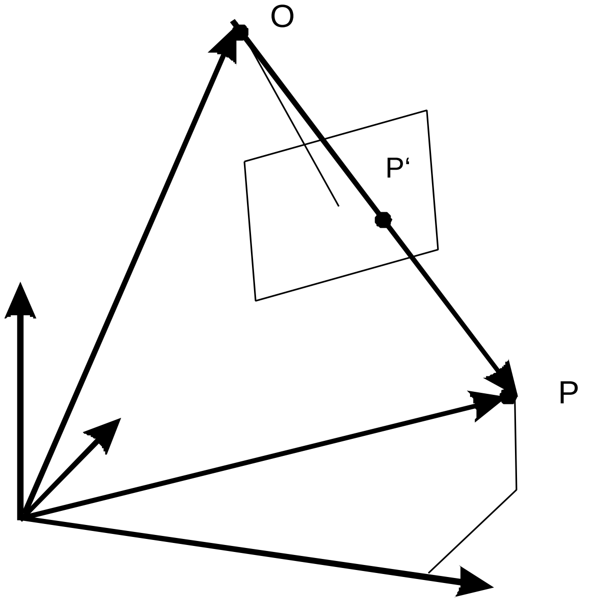


figure 5: collinearity condition: projection center O, image point P’ and ground point P are located on a straight line

$$x' = f \bullet \frac{a_{11}(X-Xo) + a_{21}(Y-Yo) + a_{31}(Z-Zo)}{a_{13}(X-Xo) + a_{23}(Y-Yo) + a_{33}(Z-Zo)}$$

$$y' = f \bullet \frac{a_{12}(X-Xo) + a_{22}(Y-Yo) + a_{32}(Z-Zo)}{a_{13}(X-Xo) + a_{23}(Y-Yo) + a_{33}(Z-Zo)}$$

formula 2: collinearity equation
f = calibrated focal length
a = coefficient of rotation matrix

The name bundle block adjustment is based on the fact that the rays from the projection center to the photo points are building a bundle of rays – this is the original information used in photogrammetry.

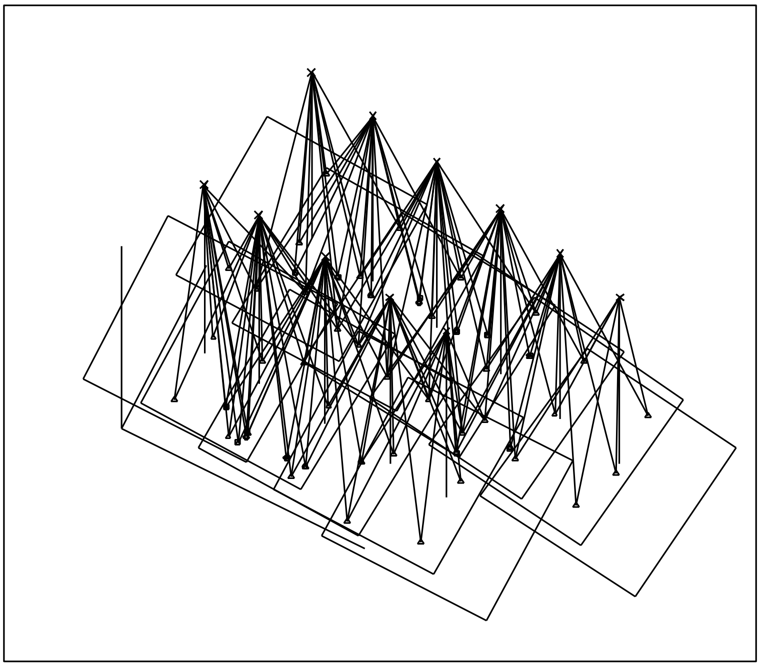


figure 6: bundle block adjustment as optimal fit of the bundle of rays
- rays are intersecting at the ground point

The bundle block adjustment is using the photo coordinates, that means, the original information which is available in the photogrammetry. Based on the same photo coordinates by this reason the bundle block adjustment is leading to more accurate results than the other methods. Of course some additional corrections like self calibration with additional parameters are improving the results as well as additional observations like GPS-positions of the projection centers.

4. IMAGE COORDINATES

For the bundle adjustment image coordinates are required, but image coordinates cannot be measured directly. The origin of the image coordinates is the principal point. The location of this point is known by the calibration in relation to the fiducial marks or the pixel address in the case of a digital camera. The instrument coordinates of the photo points together with the fiducial marks have to be available and the photo points together with the measured fiducial marks have to be transformed to the calibrated fiducial marks. Different methods of transformations can be used.

$$x' = a_1 + a_2 \bullet x - a_3 \bullet y \qquad y' = a_4 + a_3 \bullet x + a_2 \bullet y$$

similarity transformation, 4 unknowns

$$x' = a_1 + a_2 \bullet x + a_3 \bullet y \qquad y' = a_4 + a_5 \bullet x + a_6 \bullet y$$

affinity transformation, 6 unknowns

$$x' = \frac{a_{11} \bullet x + a_{12} \bullet y + a_{13}}{a_{31} \bullet x + a_{32} \bullet y + 1}$$

projective transformation, 8 unknowns

$$y' = \frac{a_{21} \bullet x + a_{22} \bullet y + a_{23}}{a_{31} \bullet x + a_{32} \bullet y + 1}$$

formulas 3: image transformations

The type of transformation has to be dependent upon the image geometry itself. The CCD-array of a digital camera is not changing the dimension, by this reason for such

images a similarity transformation is sufficient, but the shape of usual photos is changing by the time. The photos do have a shrinkage depending upon the age – the shrinkage is different in both coordinate directions and also an angular affinity may be available. In addition there is also the influence of the film development. Errors of 90µm over 230mm of an aerial film is common as well as a difference in both coordinate components in the range of 50µm. The effect of the angular affinity can reach 20 – 30µm. So there is no discussion, for photos at least an affinity transformation is required.
A higher degree of errors is usually not available, but this may be different especially for the second photo of each flight strip.

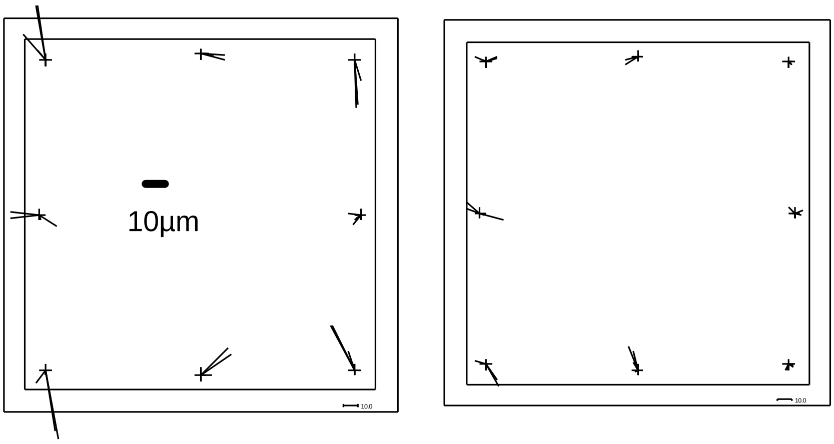


figure 7: residuals at fiducial marks of the 2nd photos of a flight strip

left hand based on affine transformation,
right hand based on projective transformation

As it can be seen on the left hand side of figure 7, perspective errors may happen – they have reached after affinity transformation a size of up to 40µm. With a projective transformation the problem can be solved, but it is better to have 2 additional images before every flight strip, so this special problem can be avoided.
Close range images are often using reseau cameras. The image coordinates should be improved by a bilinear transformation based on the discrepancies between the calibrated and measured reseau crosses.
In addition to the dimensional problems, which can be solved by the transformation to the fiducial marks, other problems are existing. The lens system may have a radial symmetric distortion, this should be respected as pre-correction in the image coordinates. Usual bundle block adjustments with aerial photos are using the national net coordinates, this does not correspond to the mathematical model because the national net is not an orthogonal coordinate system, it is following the curved earth. For aerial photos it is sufficient to respect the effect of the earth curvature and also refraction by a pre-correction of the image coordinates. Only in the case of images taken from space, the second order effects of this approximate solution cannot be accepted, the block adjustment should be made in a tangential coordinate system of the earth ellipsoid, so only the refraction has to be respected at the image coordinates. But for space images the usually used polynomial formulas are not valid, they are causing totally wrong results with corrections in the range of several millimeters instead of few microns. The formula 4 is based on the 1959 ARDC standard atmosphere correction and valid also in the space.

$$Dr = \left(0.113 \cdot \frac{(P1 - P2) \cdot Zg}{Zf - Zg} - \left(\frac{Zf \cdot 2410}{Zf^2 - Zf \cdot 6 + 250} - \frac{Zg \cdot 2410}{Zg^2 - Zg \cdot 6 + 250}\right) \cdot \left(r + \frac{r^3}{f^2}\right) \cdot 10^{-6}\right.$$

Zg = height above ground [m], Zf = flying height [m], f = focal length [mm], r = radial distance in the image [mm], P1 = air pressure in terrain height [mb], P2 = air pressure in flying height [mb], $P = e^{(6.94 - Z \cdot 0.125)}$ [Z in km]

formula 4: refraction correction of the image coordinates

Aerial cameras are calibrated in the laboratory, but this does not exactly correspond to the actual situation during photo flight. There is a vertical temperature gradient in the lens system causing different distortions and the pressure plate may not be totally flat as it should be. Deformations of the pressure plate in the range of 40µm have been seen. These and also other effects are causing systematic image errors – that means the real image geometry does not correspond to the mathematical model of a perspective image. The expression “systematic image error” is not exact, because the real image cannot have an error, it is more an error of the mathematical model, which is based on the perspective geometry.

r 2 = x2 + y2   arctan b = y/x

1. $x' = x - y \cdot P1$	$y' = y - x \cdot P1$	
2. $x' = x - x \cdot P2$	$y' = y + y \cdot P2$	
3. $x' = x - x \cdot \cos 2b \cdot P3$	$y' = y - y \cdot \cos 2b \cdot P3$	
4. $x' = x - x \cdot \sin 2b \cdot P4$	$y' = y - y \cdot \sin 2b \cdot P4$	
5. $x' = x - x \cdot \cos b \cdot P5$	$y' = y - y \cdot \cos b \cdot P5$	
6. $x' = x - x \cdot \sin b \cdot P6$	$y' = y - y \cdot \sin b \cdot P6$	
7. $x' = x + y \cdot r \cdot \cos b \cdot P7$	$y' = y - x \cdot r \cdot \cos b \cdot P7$	
8. $x' = x + y \cdot r \cdot \sin b \cdot P8$	$y' = y - x \cdot r \cdot \sin b \cdot P8$	
9. $x' = x - x \cdot (r^2 - 16384) \cdot P9$	$y' = y - y \cdot (r^2 - 16384) \cdot P9$	
10. $x' = x - x \cdot \sin(r \cdot 0.049087) \cdot P10$	$y' = y - y \cdot \sin(r \cdot 0.049087) \cdot P10$	
11. $x' = x - x \cdot \sin(r \cdot 0.098174) \cdot P11$	$y' = y - y * \sin(r \cdot 0\ 0.098174) \cdot P11$	
12. $x' = x - x \cdot \sin 4b \cdot P12$	$y' = y - y \cdot \sin 4b \cdot P12$	
13. $x' = x + x \cdot P13$	$y' = y + y \cdot P13$	= focal length or GPS shift Z
14. $x' = x + P14$	$y' = y$	= principal point x or GPS shift x’
15. $x' = x$	$y' = y + P15$	= principal point y or GPS shift y’
16. $x' = x + x \cdot tgps \cdot P16$	$y' = y + y \cdot tgps \cdot P16$	GPS drift Z
17. $x' = x + tgps \cdot P17$	$y' = y$	GPS drift x’
18. $x' = x$	$y' = y + tgps \cdot P18$	GPS drift y’
19. $x' = x + (x \cdot \cos\kappa + y \cdot \sin\kappa) \cdot P19$	$y' = y$	GPS-datum X
20. $x' = x$	$y' = y + (-x \cdot \sin\kappa + y \cdot \cos\kappa) \cdot P20$	GPS-datum Y
21. $x' = x$	$y' = y + tgps^2 \cdot P21$	
22. $x' = x - (y/f - x/r^2) \cdot P22$	$y' = y - (y/f - y/(c2 + y2)) \cdot P22$	
23. $x' = x - \arctan y/x \cdot P23$	$y' = y$	
24. $x' = x - \sin(y/300.) \cdot P24$	$y' = y$	
25. $x' = x$	$y' = y - \sin(y/300) \cdot P25$	
26. $x' = x - \sin(y/150.) \cdot P26$	$y' = y$	22 - 26 for panoramic photos
27. $x' = x - x \cdot \sin(r * 0.08)/r^{3/2} \cdot P27$	$y' = y - y \cdot \sin(r * 0.08)/r3/2 \cdot P27$	

27 - 28 for special cameras like fish eye optics

28. $x' = x - x\cdot(r^4 - 2.6843\cdot108)\cdot P28$ $y' = y - y\cdot(r4 - 2.6843\cdot108)\cdot P28$

formula 5: set of additional parameters used in program system BLUH

The stability of an aerial camera is limited, the systematic image errors do change from photo flight to photo flight, so an a priori calibration will have only a limited meaning. But within a photo flight the systematic image errors are more or less the same, that means all photos of the same flight will have the same deformation. This effect can be determined by the self calibration with additional parameters (see formula 5). Additional unknowns are included in the adjustment which are able to fit the difference between the real photo geometry and the mathematical model of the perspectivity.

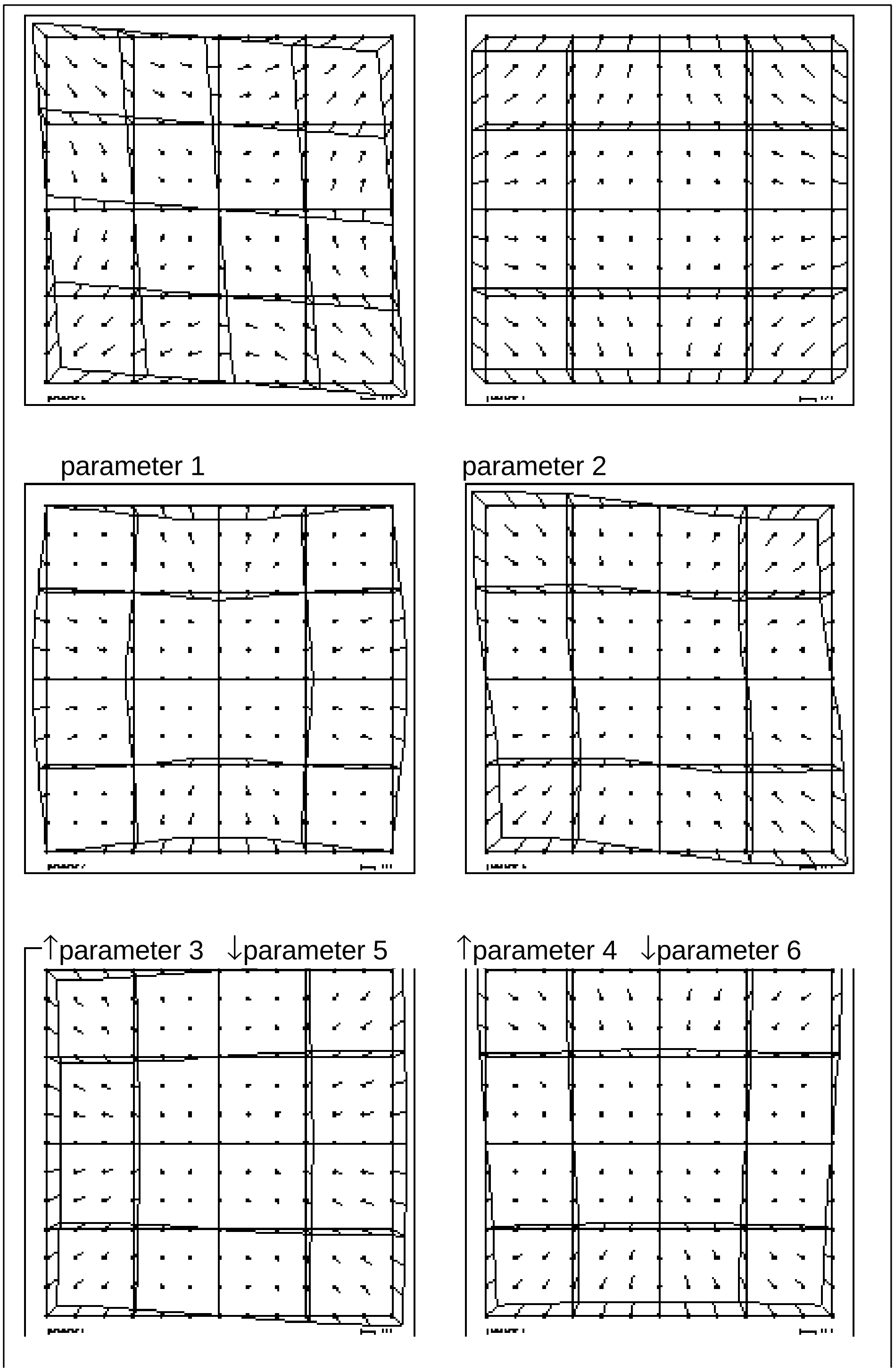
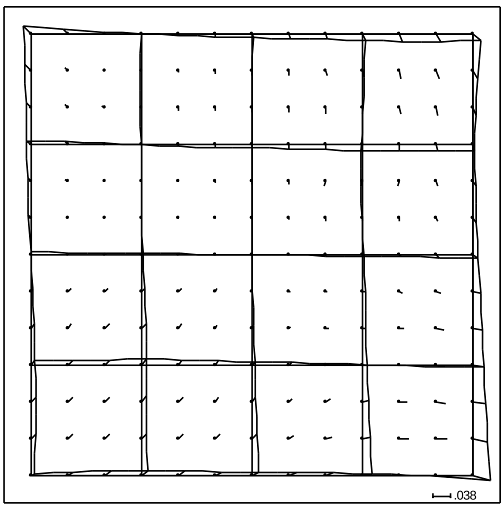


figure 8: effect of the additional parameters 1 – 6 to the image coordinates

The listed set of additional parameters are included in the Hannover bundle block adjustment program system BLUH. Also other sets of additional parameters are existing, but the listed set of formulas usually has the smallest correlation between the parameters and respects the existing physical problems. Especially the very often used polynomial expressions for fitting the radial symmetric lens distortion by the formula: $\Delta r = K_1 \bullet r^3 + K_2 \bullet r^5 + K_3 \bullet r^7$ is leading to extreme correlation between the unknowns and is only effective in the extreme image corners, by this reason it should be avoided. The parameters 9 up to 11 (formula 5) do avoid a strong correlation and can model the radial symmetric distortion in a better way.
The additional parameters can be connected to all photos of one camera or to all photos in the block (camera invariant or block invariant). The block adjustment should be done only with the required parameters. Not necessary additional parameters can worsen the computed ground coordinates. The standard deviation of unit weight of the block adjustment sigma 0 is usually smaller if the number of parameters is larger, but the size of sigma 0 is not a criteria for the possibility of the mathematical determination.
The special additional parameters (13 – 28) should only be used if they are justified. So the parameters 13 – 15 cannot be used for usual aerial photos if no GPS-values of the projection centers are available, otherwise this may cause numerical problems up to a singularity of the normal equation system. But especially in close range photogrammetry based on partial metric cameras, the inner orientation may be included in the solution.
Three different statistical tests (correlation, total correlation, Student test) should be made for checking the necessity of any single additional parameter. So at first the standard set of parameters (1-12) should be included in the adjustment and the program should reduce the set to the required parameters.
In general there is no knowledge about the systematic image errors in advance, but more or less any general structure of systematic problems with the exception of very local problems can be fitted by the self calibration with additional parameters.



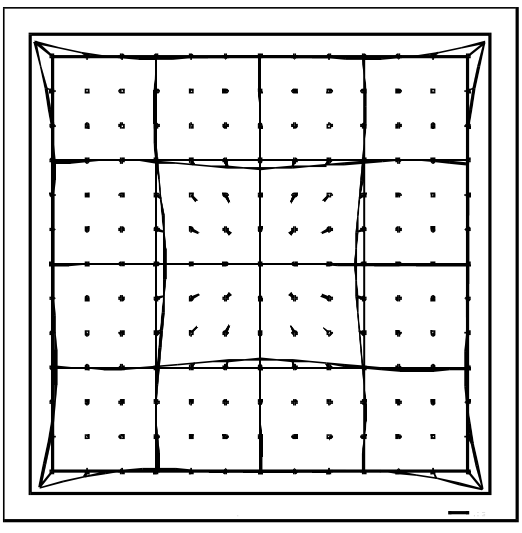


figure 9: systematic image errors determined by self calibration

So in figure 9 two examples of determined systematic image errors are shown. On the left hand side there is the effect of an aerial camera with in maximum 39µm and on the right hand side the effect of a Rolleimetric 6006 camera with a strong radial symmetric lens distortion up to 138µm. Especially the radial symmetric lens distortion can be determined without problems, so usually no pre-calibration of it is required if at least few images are included in the bundle block adjustment.

5. CONTROL POINTS

The block adjustment requires at least 2 horizontal and 3 vertical control points if no projection center coordinates are used. With GPS projection center coordinates the adjustment can be handled also without control points. For the introduction of additional parameters, 4 horizontal control points are required if no crossing flight strip is available. But these are just the theoretical limits. For reliable and accurate results more control points are necessary.

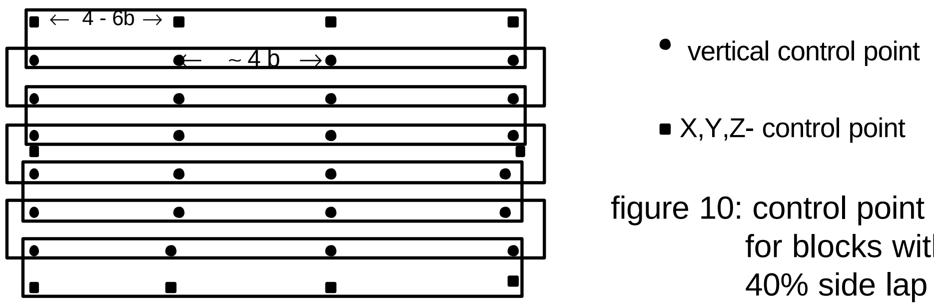


figure 10: control point distribution for blocks with 20% - 40% side lap

Complete control points are required only at the periphery of the blocks with an interval of 4 to 6 base length. No extrapolation out of the area of control points exceeding 0.5 up to 1 base length should be made. In the case of parallel flight lines with less than 50% side lap in or close to any side lap area vertical control points are required with a distance in the flight direction of approximately 4 base length. If a smaller number of control points will be used, the accuracy of the block adjustment will be less than the accuracy within a model, individually oriented by control points. In addition small blunders of the control points cannot be determined. If vertical control points are missing totally in or close to the side lap of two neighbored strips, the normal equation system can get singular - that means the block cannot be computed.

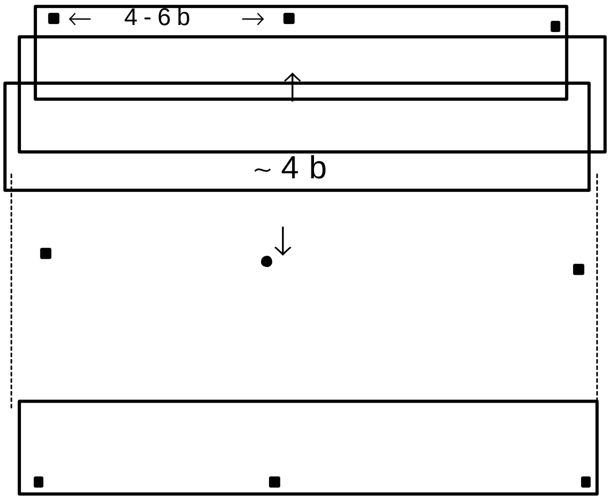


figure 11: control point distribution for blocks with 60% side lap

In the case of 60% side lap, the number of vertical control points can be reduced to an equal distance in both directions of approximately 4 base length. With 60% side lap the vertical accuracy of object points will be raised by the factor 2. In addition the reliability will be improved.

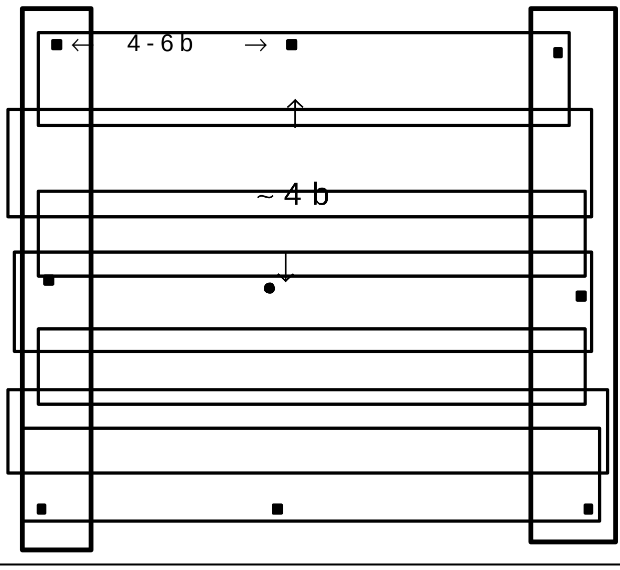


figure 12: control point distribution for blocks with crossing strips

The most economic possibility of the reduction of the number of control points is the use of control strips - crossing strips in the area of the control points. In this case the necessary distance of control points is the same like in the case of 60% side lap, but a smaller number of photos is required. Such a block configuration is strongly recommended for a combined adjustment with projection center coordinates determined by kinematic GPS positioning.

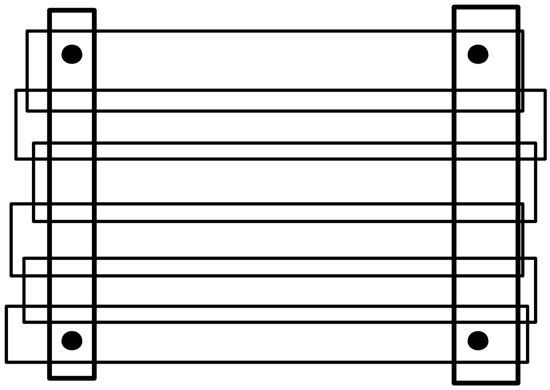


figure 13: combined block adjustment with GPS: block stabilized by crossing strips - 4 control points sufficient

● X,Y,Z-control point

figure 14: combined block adjustment with GPS: block without crossing flight strips

● X,Y,Z-control point

o Z-control point

For a combined bundle block adjustment with projection center coordinates determined by relative kinematic GPS positioning, control points are required only in the block corners if the flight lines do not exceed 30 base length. For a sufficient reliability, 2 control points should be located in each corner. If cycle slips of the GPS-positioning cannot be avoided, 2 crossing flight or a line of vertical control points at both sides of the blocks are required (see figure 13 and 14).

6. COMBINED BLOCK ADJUSTMENT WITH GPS

The ground survey of control points is very time consuming and expensive. Sometimes it is more expensive than the photo flight together with the block adjustment. A possibility to reduce the number of required control points is the use of precise information about the projection centers.

The NAVSTAR Global Positioning System (GPS) is a satellite based radio navigation system which allows a worldwide precise three-dimensional positioning based on distances from the satellite to the ground station and the antenna in the aircraft. By theory the distances to 3 satellites are sufficient but for the correction of the receiver clock an additional satellite is required.

The GPS-satellites are using 2 different frequencies, L1 with 19cm and L2 with 24cm wavelength (carrier). In addition there is a modulation with 2 different codes - the P-code with 29m which has been changed to the classified Y-code and the C/A-code with 293m wavelength. The distances to the satellites are determined by phase measurements. A phase can be determined with approximately ±0.5% of the wavelength. The CA-code can be improved by carrier phase smoothing up to an accuracy of +/-0.2m. So by theory a very precise positioning is possible if the satellite positions are known accurate enough. Since the end of the selective availability (SA) so the on-line navigation can reach an accuracy of approximately ±10m. An improvement of this situation is possible by relative positioning. In the same area all positions are influenced in the same way especially by effects caused by the ionosphere. If a reference station is located on a known position, the moving receiver in the aircraft can be determined in relation to this. Absolute GPS-positions (without reference station) are not accurate enough for the combined block adjustment.

Based on carrier phase measurements the relative kinematic positioning can be done in the aircraft with a relative accuracy up to ±4cm. But it is not easy to solve the ambiguities (the determination of the number of waves belonging to the measured phase). Especially during the turn around from one flight strip to the next the reference to some satellites can be lost, causing a so called cycle slip. This is resulting in constant errors of the positions (shifts), sometimes also in higher order errors which can be eliminated in the first degree by time depending parameters (drifts) changing from flight strip to flight strip. A simple relative positioning with the C/A-code is usually limited to ±1.5m in the 3 coordinate components, so it can only be used for topographic mapping if this accuracy is sufficient. A carrier phase smoothing can result in GPS-positions with an absolute accuracy of ±0.2m. Such a positioning is also possible close to on-line if a broadcasting reference station is used. The C/A-code has the advantage that it is by theory not affected by ambiguity problems - this is only belonging to the carrier phase, but also changing systematic errors of CA-code data have been seen.

Inertial measurement Unit (IMU) data also can be handled for a support of the bundle block adjustment in the case of single flight strips. A support only by GPS-data of the projection centers for single flight strips only can extend the distance between control points up to 10 base length (without GPS only 4 base length). In the case of a larger control point interval, the strip can rotate around the line of the projection centers. This can be avoided by IMU-data.

The GPS-positioning is based on the world geodetic system WGS84. A coordinate transformation to the national net coordinates has to be done. In addition to the transformation itself also the datum transfer has to be respected if this is not indirectly be done with the reference point. In addition the Geoid undulations have to be respected, the point heights in the WGS84 are ellipsoid heights and not heights in the national coordinate system which are related to the Geoid. With a local transformation to control points, the effect of the Geoid can be minimized.

The positions of the projection centers cannot be determined directly. Only the antenna positions are recorded in a constant time interval. Based on the recording of the instant of exposure an interpolation of the antenna position at the time of exposure has to be done. From this position the offset to the projection center must be respected. The offset in relation to the camera frame, respecting the location of the projection center (the distance from the frame to the entrance node is larger than the focal length), has to be measured and respected. A transformation of this vector to the national net coordinate system can be done in the bundle block adjustment. But this is only possible if the camera is not rotated during photo flight in relation to the aircraft, but usually this will be done for drift compensation. The relative rotations should be recorded and respected. If this will not be done, the camera rotation should be the same in a flight strip. Under such a condition there are mainly constant errors which can be determined by additional parameters individually for every strip. Only if an IMU is attached to the camera body, the offset can be respected on-line. There is no influence of the drift-rotation to the offset, if the antenna is mounted directly vertical above the camera.

The constant and also time and location depending errors of the projection centers in the case of carrier phase measurements may be different from flight strip to flight strip, but changing systematic errors have been seen also with CA-code data.
The determination of 3 or 6 additional unknowns for every strip is only possible with a higher number of control points which should be avoided or with at least 2 crossing flight strips. With 2 crossing strips the combined block adjustment can be handled with just 4 control points (one in any corner) also with 6 additional parameters for every flight strip (see figure 13 and 14).

7. BLUNDER DETECTION

The blunder detection can be done in two steps. The first search should be done during the calculation of approximate photo orientations for the block adjustment and after this in the block adjustment itself. A manual search for blunders is very time consuming, in addition not in any case the blunder is behind the observation with the largest correction. The correction (residual) of an observation is not identical to the true error caused by the least squares adjustment. This problem can be solved by 2 different methods for automatic error detection, the data snooping and the robust estimators.

7.1 DATA SNOOPING (BAARDA METHOD)

The partial redundancy **r** is the relation between the correction **vi** and the original error **si** of an observation.

$$v_i = -r_i \cdot s_i$$

formula 6: relation of correction vi to the true error si

formula 7: partial redundancy (redundancy number)

$$r_i = (Q_{ee}\ P_{ee})_{ii}$$

$$Q_{vv} = Q_{ee} - A \cdot N^{-1} \cdot A^T$$

$$x = N^{-1} \cdot A^T \cdot P \cdot f$$

Q_{ee} = cofactor matrix of observations
Q_{vv} = cofactor matrix of unknowns
P_{ee} = weight matrix of observations
N = normal equation matrix
A = matrix of coefficients
x = unknowns
f = observations

In the case of a relative orientation, the partial redundancy r in the model corner can be much smaller than in the center. In general the partial redundancy must be respected for the identification of blunders if the redundancy numbers are quite different like in the case of a relative orientation.

$$w_i = \frac{v_i \cdot \sqrt{P_i}}{\sqrt{r_i} \cdot sigma}$$

formula 8: normed correction

wi = normed correction
vi = correction (e.g. y-parallax)
Pi = weight of observation i
sigma = standard deviation of unit weight

nabla = vi / ri	formula 9: possible blunder, which can be included

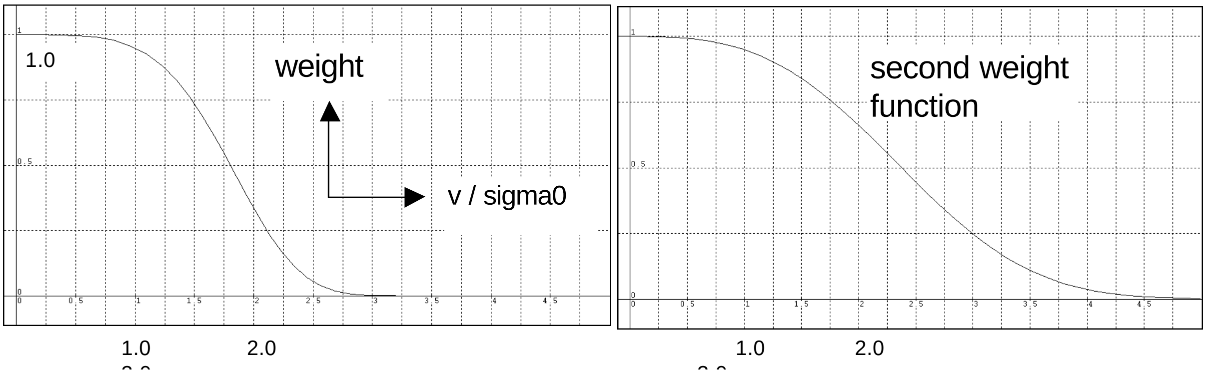
7.3 ROBUST ESTIMATORS

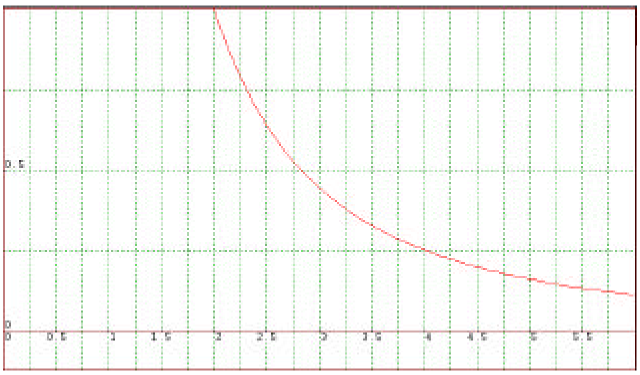
Robust estimators are iterative weight manipulations of the observations. The robust estimators will modify the weights of the observations (e.g. photo coordinates) depending upon the size of the residuals. By this method, the weights of incorrect photo coordinates will be reduced until there is no more influence of the blunders to the adjustment. The time consuming manual elimination of blunders will be avoided and the program user must not have intensive experience in manual error detection. The weight of the observations (photo coordinates) can be modified by following formula:

$$p = e^{-REA \cdot (r/sigma0)**REC}$$

p = weight
r = maximum of the absolute values of the residuals of the x- and y-photo coordinates
sigma0 = standard deviation of unit weight of the preceding iteration

formula 10: weight function for robust estimators





special weight function („S“)

```
for V < 2 * SIGMA0 : P=1.0

for V > 2 * SIGMA0 :

        SIGMA0 * SIGMA0 * 4
P =    -------------------
              V * V
```

figure 15: weight functions of the robust estimators used in program system BLUH

In program BLUH there are 2 possibilities for the robust estimators. The upper formula for the weight function, also called Danish method requires a more stable block with good image connections. It uses for the first iteration with robust estimators the factor REC = 4.4. The last iteration will be computed with REC = 3.0. REA will be identical to 0.05. Sigma0 will have the value of the preceding iteration, but it will not be smaller than 5.0 microns. The in the lower part shown weight function can be used also for not so stable blocks, but it requires usually more than one program run if a combination of larger and smaller blunders are included in the data set.
Usually 2 - 4 iterations with robust estimators are sufficient. A too high number of iterations can cause numerical problems especially for the upper method, if the weights for observations, which are necessary for connections are becoming too small. This can happen especially for control points. The lower method shows more stable results.

7.3 HANDLING OF BLUNDER DETECTION

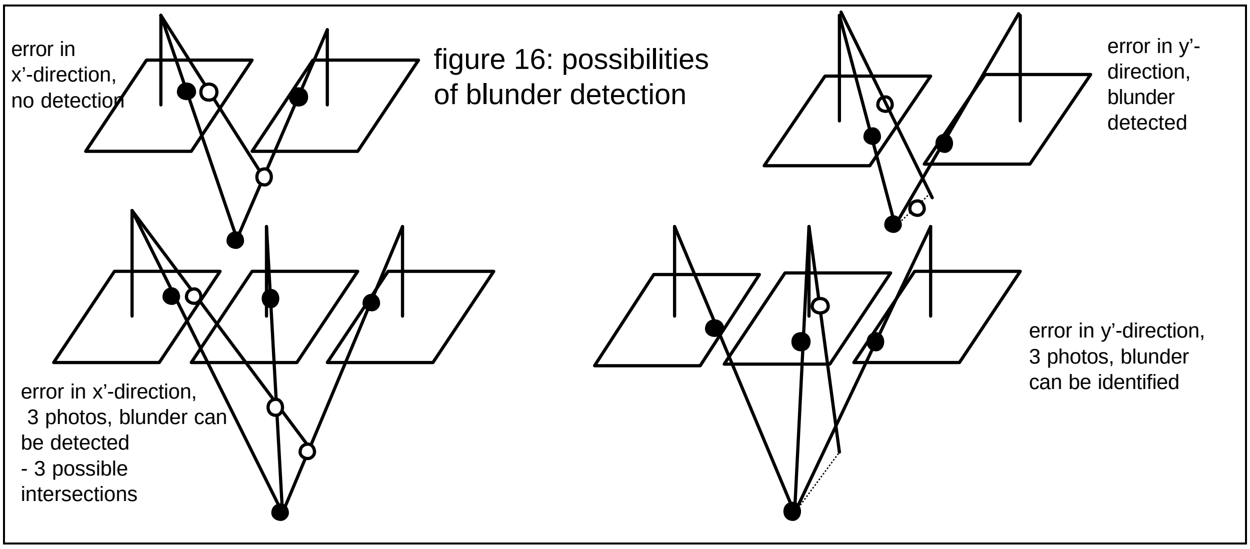


figure 16: possibilities of blunder detection

If a grossly incorrect point is measured only in two photos, no discrepancy can be seen if the blunder is located in the base direction (see figure 16, upper left). Blunders across the base direction can be detected because the rays will not intersect (see figure 16, upper right). That means, points used just in two photos can be accurate but they are not reliable.
For the blunder identification one more observation than for the blunder detection is required. In the case of a blunder in the base direction, with 3 photos the blunder can be detected (see figure 16, lower left) but three intersections are available - any can be correct - one more observation is required for the correct identification of the blunder. In the case of a blunder across the base direction with 3 observations, two will have an intersection, so the blunder can be identified (see figure 16, lower right).
The data snooping is a powerful tool for the identification of blunders, but by theory it is only able to identify one blunder at the time. The correct identification of more blunders is

very difficult in one run. By this reason the data snooping has some advantages for the calculation of approximate photo orientations where the computation time is short and one blunder after the other can be eliminated automatically. For the bundle block adjustment itself, the robust estimators are better, they can identify a higher number of blunders in the same iteration.
Even with robust estimators not all blunders can be detected in one program run if large and small blunders are mixed because large blunders can cause deformations of the block which will not go back totally during iteration with robust estimators.

8. APPROXIMATION OF ACCURACY

If a sufficient number of control points and tie points are available, at least the accuracy reached by an individual model orientation is guaranteed.

horizontal standard deviation
SX = SY = photo scale number • δo

δo = standard deviation of unit weight (from BLUH)

vertical standard deviation
SZ = photo scale number • h / b • spx
h = flying height above ground
b = photo base (distance of neighbored projection centers)
spx = standard deviation of x-parallax

The height to base relation h/b is identical to f/b'
f = focal length
b' = photo base in photo = (1-p) • s'
p = endlap
s' = image size in flight direction (in case of aerial images = 230 mm)
In case of the usual endlap of 60%, the photo base in the photo is identical to b'=92mm
In case of wide angle (c=153mm) the height to base relation is identical to h/b = 1,6 in the case of normal angle (c=305mm) it is identical to h/b = 3,2

In the case of a monoscopic photo measurement, the accuracy of the x-parallax is reaching spx=δo • 1,4, in the case of a stereoscopic measurement it is approximately identical to δo.
That means, the Z-components will have a higher standard deviation than the X- and Y-component, at least by the height to base relation.
The estimation is valid for well defined natural or targeted points. In the case of artificially marked points (PUG, Transmark ...) it is less accurate up to a factor of 2. Because the artificial marked point in one image is disturbing the setting of the floating mark to the corresponding point in the other image.
For usual height measurements in a model only an accuracy in the image of spx=+/-10µm will be reached because of a lower contrast of regular distributed points in relation to control points. This is corresponding to approximately SZ=0,1%o of the flying height above ground.

example f = 153mm, image scale number = 12 000 corresponding to flying height above ground hg= f • scale number = 1836m , that means SZ= 0,1%o of the flying height = +/-18cm.

9. PROGRAM SYSTEM BLUH

The described methods are realized in the Hannover program system BLUH for bundle block adjustment. The program system BLUH is an extensive system of programs well grounded on statistics with a high degree of automation. No estimates of the unknowns are required. Blunders are identified by the method of data snooping and robust estimators. Extensive pre- and post processing do enable the handling also of special data sets with not perspective geometry. Graphic representations are simplifying the data analysis and do give an overview over geometric weak parts of the block.
Because of the rigorous mathematical model and the self calibration only few control points are required. In the center of the block horizontal control points are not required and with crossing flight strips the number of vertical control points can be minimized.

10. EXPERIENCES IN BUNDLE BLOCK ADJUSTMENT

Today usually no photogrammetric job will be done without block adjustment. The bundle block adjustment is a very flexible tool, which can be used for the determination of the photo orientations and ground coordinates, but also for the camera calibration. In some cases, especially for high precision, the adjusted ground coordinates may be the final result. An example of this is the determination of subsidence in a coal mining area. Based on a photo scale of 1 : 4000, 60% end lap, 60% side lap and a crossing flight with 20% side lap, that means three times the number of required photos, the positions of man holes can be determined with an accuracy of SX=SY=+/-2cm and SZ = +/-3cm.

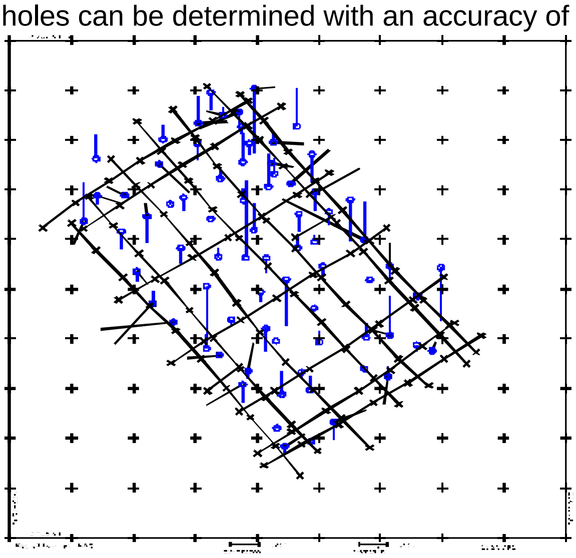


figure 17: precise determination of points by bundle block adjustment
p=60%, q=60% + crossing flight
SX = SY = +/- 2cm, SZ = +/- 3cm

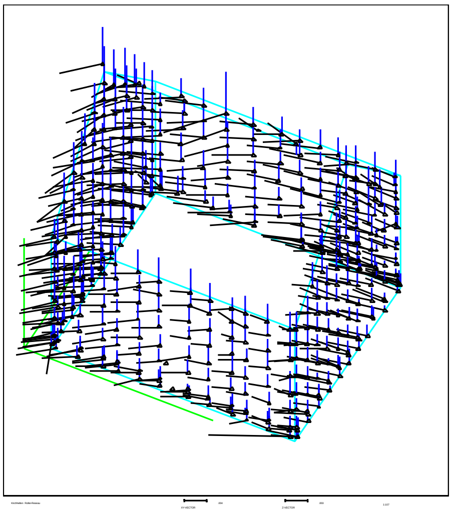
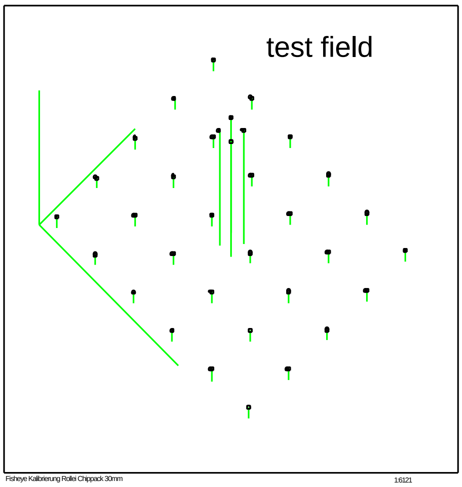


figure 18: determination of a building for deformation control with photos taken from all sides which was leading to an accuracy of targeted points with a standard deviation in all three components better than +/-1mm.



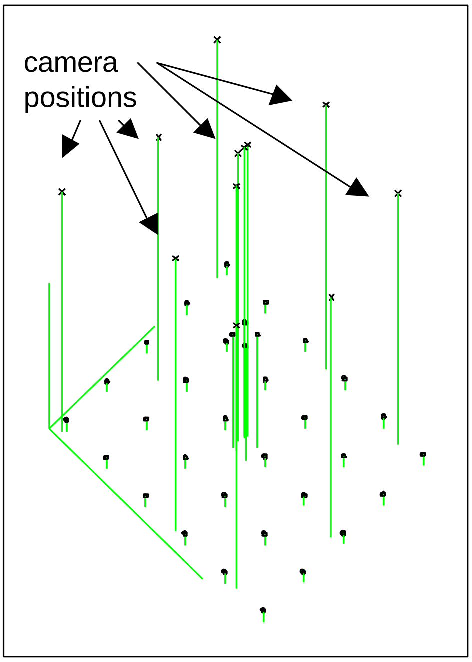


figure 19: calibration of close range cameras

The calibration of close range cameras can be made with several photos taken from the same test field from different directions. The targeted points of the test field must not be known in advance, they can be determined within the bundle block adjustment. With the digital cameras DCS 420 and the Rolleimetric Chip-Pack an accuracy of +/-0.02 up to +/-0.03 pixel have been reached also with a fish-eye lens system.

The quality of the combined bundle block adjustment is depending upon the quality of the GPS-values, which may be quite different. Systematic errors of the GPS-values have to be expected.

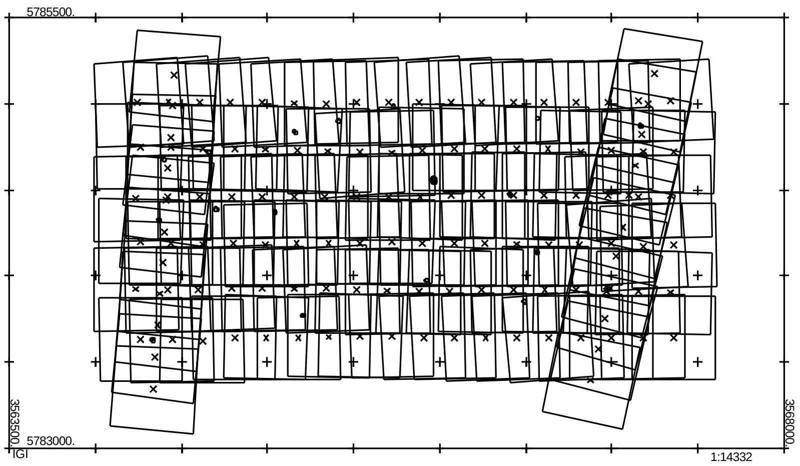


figure 20: block Hildesheim – combined adjustment without control points

In figure 20, the block Hildesheim is shown with the typical configuration for combined bundle blocks with projection center coordinates determined by kinematic GPS-positioning. In this case, the GPS-positioning was supported by a inertial measurement unit (IMU) – a combination of 3 giros with 3 accelerometers. This combination avoids cycle slips. With a broadcasting local GPS-reference station the GPS-positions have been determined directly during photo flight. No large systematic errors have been available, so it was possible to handle the Hannover bundle block adjustment program system BLUH without control points.

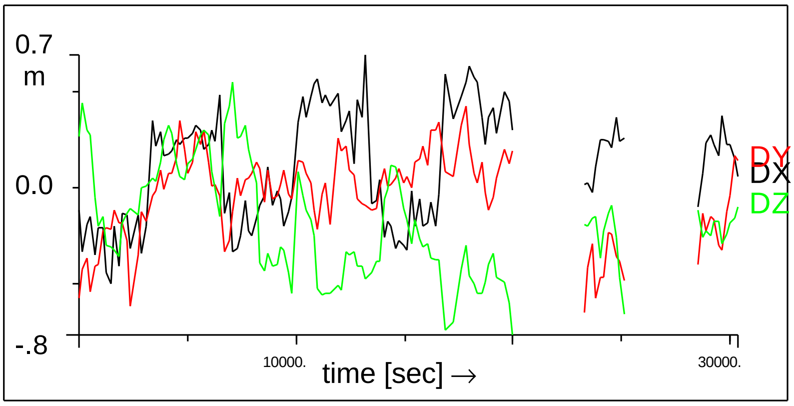


figure 21: block Hildesheim discrepancies at the projection centers, GPS - controlled bundle adjustment

	control points	sigma0 [µm]	SX [cm]	SY [cm]	SZ [cm]
reference no GPS	18	13.5	1.6	2.5	2.8
minimal control no GPS	4	13.6	3.4	5.2	48.2
GPS +	4	13.5	2.9	3.7	14.0
GPS +	1	16.8	6.6	9.0	17.1
no control points	0	19.3	**12.3**	**17.2**	**19.4**

table 1: accuracy of independent check points determined by combined block adjustment with and without control points

Of course, as it can be seen in table 1, the bundle block adjustment of this block with an image scale of 1 : 2000 together with GPS and control points is leading to better results, but it was possible to reach without control points a ground accuracy, determined at independent check points, better than +/-20cm in all 3 coordinate components. Without GPS and just with 4 control points, the vertical accuracy has not reached this level. This is a typical result, GPS supports especially the vertical accuracy in the case of a block structure. In the case of long and only few flight strips, GPS is also required for the horizontal accuracy.

Special problems have to be solved in the case of blocks with data acquisition by automatic aerial triangulation, that means, automatic determination of the tie points. These blocks include a very high number of points per photo – up to 3000 points in a photo have been used, in addition a very high number of blunders is included – up to 15 000 blunders have been in a block of 439 photos with 78 000 ground points, so the blunder determination has to be done automatically. The sigma 0 of such blocks is usually limited to approximately +/-9µm. But caused by the high number of points, the inner accuracy of the photo orientations is better than in manually measured blocks. This should be handled very careful because several tests have been shown that there is a discrepancy to the absolute accuracy of the orientations. Systematic errors have caused that the photo orientations determined by manual photo measurements are still more accurate.

11. HANDLING OF SPACE IMAGES

In general perspective images taken from space can be handled in a usual block adjustment, but we do have some special conditions. The earth curvature cannot be respected as a correction within the images, the influence is too large, so second order effects cannot be neglected. Especially the height will be influenced. Also the geometric change by the map projection is causing problems. Because of this. the space images should be handled in an orthogonal coordinate system. For a better separation between the horizontal and the vertical coordinate components a coordinate system in a tangential plane the earth ellipsoid has some advantages. In program system BLUH the coordinates can be transformed with program BLTRA.

The influence of the refraction to the image coordinates (formula 4) is small for images taken from space. Most of the existing formulas for refraction correction are polynomial. These formulas usually are limited to a flying height up to 10 – 15km and are totally wrong for space applications.

The line scanners like SPOT, IRS-1C/1D and MOMS do have the perspective geometry only in the sensor line. In the direction of the orbit it is close to a parallel projection. So the photo coordinates as input for the colinearity equation are simplified to **x' = (x', 0, -f) or (0,y',-f) -** the photo coordinate y' or x' is identical to 0.0 (by theory up to 50% of the pixel size can be reached). The pixel coordinates in the orbit-direction of a scene are a function of the satellite position, or reverse, the exterior orientation of the sensor can be determined depending upon the image position in the orbit-direction. With the traditional photogrammetric solution the exterior orientation of each single line cannot be determined. But the orientations of the neighbored lines, or even in the whole scene, are highly correlated. In addition no rapid angular movements are happening.

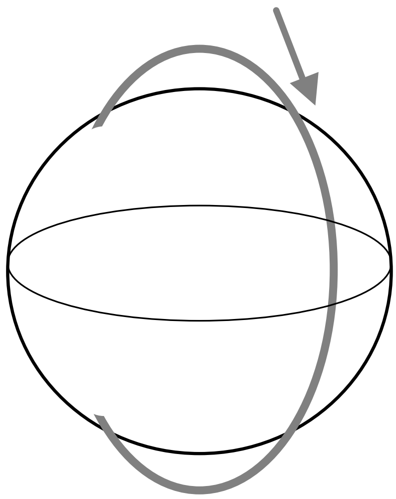


figure 22: Projection center = function of scene coordinate,

colinearity equation in sensor line across orbit – in orbit direction image coordinate = 0.0

Exterior orientation depending upon position in orbit = function of image coordinate in orbit direction

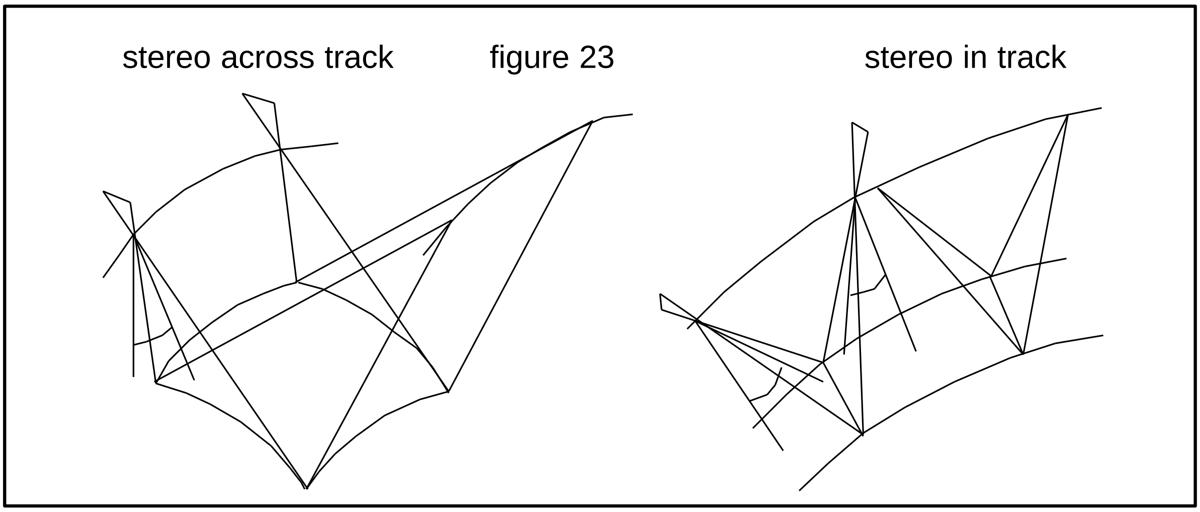


figure 23

mathematical model used in BLASPO (program system BLUH)

A fitting of the exterior orientation by an ellipse fixed in the siderial system - the earth rotation has to be respected - is used. This has been shown as sufficient for the SPOT sensor also over larger distances. In the OEEPE test area Grenoble by this method a combination of 4 neighbored SPOT scenes over a distance of 200km could be oriented with just 4 control points (in the orbit direction 200km distance between the control points) with an accuracy in the height of ±4m. The simplified mathematical models used in some other programs have to use more control points.

The header data of digital SPOT scenes are including detailed information about the actual satellite orbit. In a first program version of BLASPO the orbit was determined in fitting the ephemeris by an ellipse. The achieved results of the SPOT orientation based on this have been total sufficient. But very often no header data were available in the case of recorded images. By this reason a simplified method was developed.
The image is rotated depending upon the known view direction. With at least 3 control points and a general information about the inclination, the semi-major axis and the eccentricity of the satellite orbit, the actual trace can be computed without any actual information of the ephemeris based on an affinity transformation. The remaining errors of the mathematical model, especially the affinity and angular affinity have to be fitted by additional unknowns (additional parameters) in the image orientation. By this method errors of the exterior orientation caused by an inaccurate orbit or irregular movements within the orbit can be identified and respected. This is in general the same solution like with the discrepancies of photos against the mathematical model of perspecitivity. The respecting of the accelerations included in the SPOT-header-data has not improved the results.
With such approximate information a bundle orientation of SPOT images was possible without loss of accuracy against the use of the actual ephemeris and with the above mentioned precision.
The solution for SPOT-images is based on geometric uncorrected SPOT images, e.g. the so called **level 1A** product. For any other SPOT product, the geometric corrections have to be used as a pre-correction if it cannot be fitted by the additional parameters.

Image coordinates

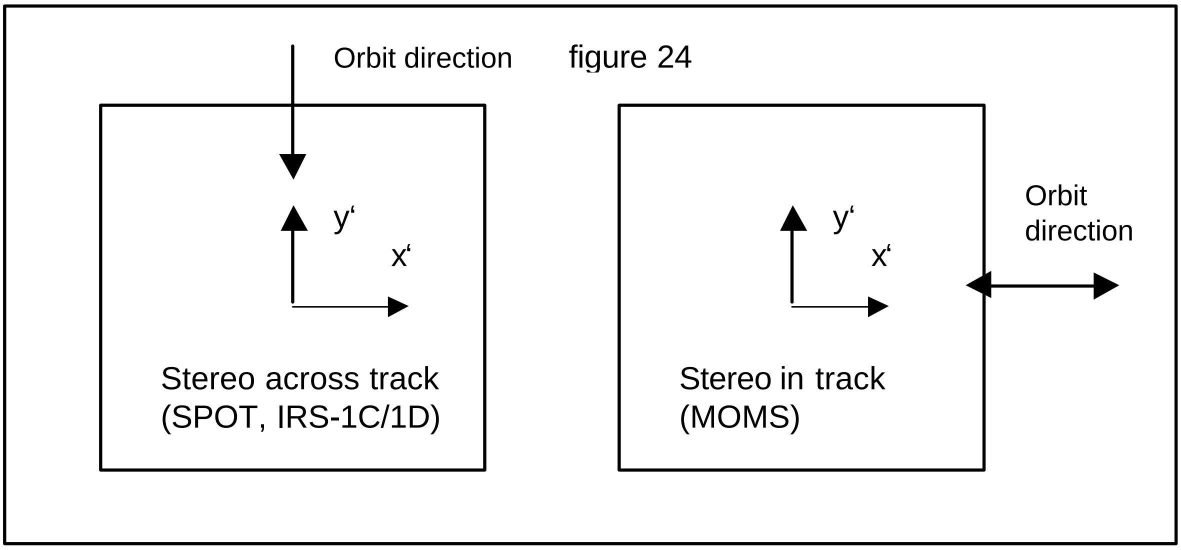


figure 24

For IRS-1C / 1D-PAN-data, special additional parameters are required because the camera includes 3 CCD-line-sensors and the geometric relation between these are not stable.

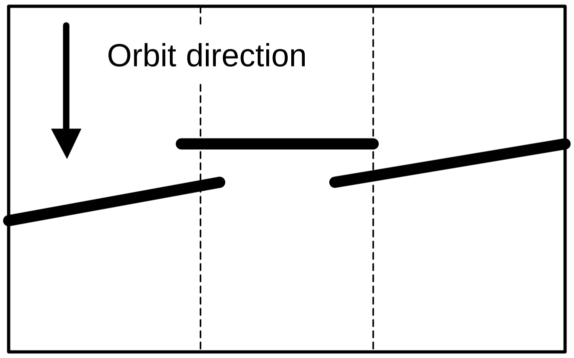


figure 25: horizontal location of IRS-CCD-line-sensors in the image plane

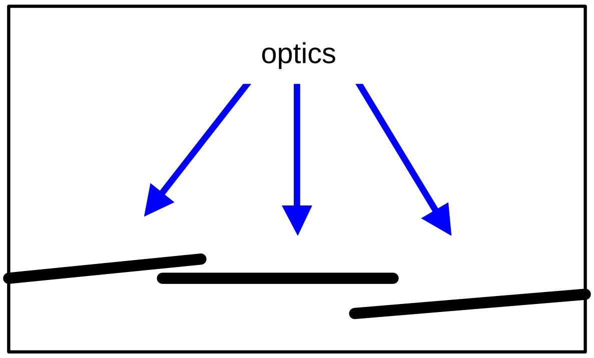


figure 26: vertical location of the IRS-CCD-lines in the image plane

The shift of the IRS-1C /1D-CCD-line sensors in the orbit can be respected by a time shift, or remaining errors by a shift of one scene to the other. A horizontal rotation against the reference CCD-line (figure 25) must be corrected by a resampling or an improved mathematical model of the block adjustment and/or the model handling. A vertical rotation and also a different focal length (figure 26) will cause a scale change in the x-direction (direction of sensor lines) of the outer scenes in relation to the reference scene in the center. There is no influence to the y-direction (orbit direction), a discrepancy of the focal length will only cause an over- or under-sampling.

```
 1  Y = Y +  P1 * Y
 2  X = X +  P2 * Y
 3  X = X +  P3 * X * Y
 4  Y = Y +  P4 * X * Y
 5  Y = Y +  P5 * SIN(Y * 0.06283)
 6  Y = Y +  P6 * COS(Y * 0.06283)
 7  Y = Y +  P7 * SIN(Y * 0.12566)
 8  Y = Y +  P8 * COS(Y * 0.12566)
 9  Y = Y +  P9 * SIN(X * 0.04500)
10  X = X + P10 * COS(X * 0.03600)
11  X = X + P11 * (X-14.)     if x >  14.
12  X = X + P12 * (X+14.)     if x < -14.
13  Y = Y + P13 * (X-14.)     if x >  14.
14  Y = Y + P14 * (X+14.)     if x < -14.
15  X = X + P15 *SIN(X * 0.11)*SIN(Y*0.03)
```

table 3: additional parameters of program BLASPO

The special problems of the IRS-1C / 1D-data can be handled by the special additional parameters 11 – 14 of table 3.

12. CONCLUSION

Today a block should be computed by a bundle block adjustment with self calibration by additional parameters. The bundle block adjustment is the most rigorous and flexible method of block adjustment. The computation with self calibration by additional parameters leads to the most accurate results of any type of block adjustment. Even based on the same photo coordinates an independent model block adjustment cannot reach the same quality; this is due to the data reduction by relative orientation, the comparatively inexact handling of systematic image errors and the usual separate computation of the horizontal and the vertical unknowns. In addition the photo orientations are required for several purposes and the other methods are not delivering this information. The bundle block adjustment is a very powerful tool, but good results only can be achieved if the input data are without problems and if a qualified program will be used. There are several bundle block adjustment programs on the market which are not qualified – by the operator support, by the support of the automatic blunder detection, by the self calibration and by the possibility of an analysis of the achieved results. In addition some programs do need a computation time in the range of hours while this can be done also in few seconds.

13. REFERENCES

Elias, B. 1998: Bündelblockausgleichung mit INS-Daten, diploma thesis University of Hannover 1998

Jacobsen, K. 1980 a: Vorschläge zur Konzeption und zur Bearbeitung von Bündelblockausgleichungen, doctor thesis University of Hannover 1980

Jacobsen, K. 1980 b: Attempt at Obtaining the Best Possible Accuracy in Bundle Block Adjustment, ISP Hamburg 1980 and Photogrametria 1982, p 219 - 235

Jacobsen, K. 1991: Trends in GPS Photogrammetry, ASPRS Baltimore 1991

Jacobsen, K. 1997: Operational Block Adjustment without Control Points, ASPRS Seattle 1997

Meier, H.-K. 1970: Modellvorstellungen zur Luftbildaufnahmedisposition und ihr Vergleich mit praktischen Testergebnissen, BuL 1971, pp 107 ff

Schmid, H. 1959: Eine allgemeine analytische Lösung für die Aufgabe der Photogrammetrie, BuL 1958, pp 103 ff