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The Radial Topology Algorithm—A New Approach for Deriving 2.5D GIS Data Models

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Abstract In this paper a new method for the combination of 2D GIS vector data and 2.5D DTM represented by *triangulated irregular networks (TIN)* to derive integrated triangular 2.5D object-based landscape models (also known as *2.5D-GIS-TIN*) is presented. The algorithm takes into account special geometric constellations and fully exploits existing topologies of both input data sets, it "sews the 2D data into the TIN like a sewing-machine" while traversing the latter along the 2D data. The new algorithm is called *radial topology algorithm*. We discuss its advantages and limitations, and describe ways to eliminate redundant nodes generated during the integration process. With the help of four examples from practical work we show that it is feasible to compute and work with such integrated data sets. We also discuss the integrated data models in the light of various general requirements and conclude that the integration based on triangulations has a number of distinct advantages.

Keywords multi-dimensional data modelling · triangulations · DEM/DTM · algorithms · performance · GIS

1 Introduction

1.1 Motivation

A focus of current research in geographic information science is the extension of traditional 2D data structures to incorporate height information [17]. The integration

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of digital terrain models (DTM) and derived products into geographic information systems (GIS) offers improved and extended analysis capabilities. As an example for the benefits of an integrated analysis, imagine the computation of a road slope along its axis. Such an information is of interest to cyclists or haulage contractors. Also, parcel based relief analysis, relevant in agriculture, poses problems to current GIS. Perhaps more important are applications in which simulations based on terrain data are carried out. In flood management, for example, one may want to know which roads are actually flooded at a given water level. Similar issues arise in urban sound simulation or when the trajectory of clouds of polluted gases needs to be computed. Another emerging application lies in car navigation where the across track slopes start to play a major role in safety systems. Also, data visualisation is sometimes hampered by the separation of 2D GIS data and DTM.

Although many of these questions can be answered using today's GIS software packages by temporarily combining DTM data with 2D data residing in different databases, the separation of planimetric and height information does not lend itself to a comprehensive and comfortable analysis. Therefore, a considerable amount of time and expertise is required to obtain the desired results. To ease and sometimes make possible a combined analysis it is thus advantageous to work with one integrated data set only, i.e., with a tesselation of the landscape into objects which "know" their shape of (or on) the terrain surface.

The emphasize of the research reported in this paper lies on *integrated 2.5D GIS datastructures* which leads to landscape objects integrated or enriched with one height value for each position in the horizontal plane. The restriction to 2.5D as compared to 3D data structures which allow for a multitude of height values at each horizontal position has several reasons. First, 3D data structures were already discussed by numerous authors, e.g., working on 3D city models or 3D geological models [5],[6]. More importantly, DTM existing in national core geospatial databases provide a valuable source for many applications, but are only 2.5D. It is the intention of this article to discuss procedures which use available data sets as input, allowing for upward compatibility of these data with new methods and thus saving investments that have already been made.

Approaches of integrated 2.5D modelling may be subdivided into different categories. A well known classification is given by Fritsch [16]. He distinguishes three different approaches of integration:

1. *Height attributing* provides a z coordinate for each node;
2. *Terrain information systems* provide for an integration of DTM functionality and possibly special data structures into a GIS without an integrated data model;
3. *Total integration* is an approach which leads to models fulfilling the requirements of integrated modelling; which is the main focus of this paper.

Although it is commonly accepted that height attributing as described above does not meet the requirements of integrated modeling, it may be discussed in a broader sense. Whereas according to Fritsch [16] a z coordinate is only provided for each *node in the plane*, a commonly used approach to link 2D objects with height information is to set up a height step model of the relief, thus, to provide a z coordinate not to nodes but to *area objects* created on the basis of contours. This leads to *object-based height attributing* instead of *node-based height attributing*. Linking landscape objects with height information is conducted by computing a map overlay. The result is an object-based "wedding cake" approximation of the landscape (the name originates from the discretization of the terrain relief by contour lines) and has initiated some interest in GIS (see also below).

1.2 Some Background

In the following it is presumed that the reader is familiar with (constrained) DELAUNAY-triangulations (DT and CDT, respectively) as well as digital terrain modelling based on triangular irregular networks (TIN). Introductory reading on these topics may be found in standard GIS text books (e.g., [26]) or computational geometry (e.g., [2], [28]). A survey on triangulations is provided by Bern and Eppstein [3], they also provide references for constrained triangulations.

One particularly important algorithm is the *incremental insertion of points into an existing TIN* (e.g., [11],[19]). The major difference between the various approaches for incremental insertion is the employed spatial access method to find the location of insertion into the TIN. Spatial access methods in TIN are treated by Devillers et al. [12].

A sound theoretical background for the topics discussed in this paper is given by the concept of *simplicial complexes* [5],[30]. In each dimension, there exists a minimum object, called *simplex*. Figure 1 shows the *0-simplex*, which is a node; the *1-simplex*, which is an edge; and the *2-simplex*, which is a triangle. Each n-simplex consists of $(n+1)$ geometrically independent simplexes of dimension $(n-1)$. A *facet* or *face* of a simplex is every simplex which is part of the first (higher dimensional) simplex. A *0-face* of a *2-simplex* is each of its 0-simplexes, i.e., corner point of the triangle, and an edge of a triangle is a *1-face*. A *simplicial complex* is a finite set of simplexes satisfying the following two properties (cf. [36]): (1) A face of a simplex in a simplicial complex is also in the simplicial complex. (2) The intersection of two simplexes in a simplicial complex is either empty or also part of the simplicial complex. An example for 2-simplicial complexes are TIN.

The *conceptual data model* of an integrated 2.5D-GIS-TIN model is illustrated by Fig. 2 using the *unified modelling language* (*UML*, Booch et al., [4]). An area object is decomposed into a set of triangles and is hence represented by a 2-simplicial complex whereas a linear object is represented by a 1-simplicial complex, i.e., it consists of a set of edges. Similarly, a point object is represented by a node.

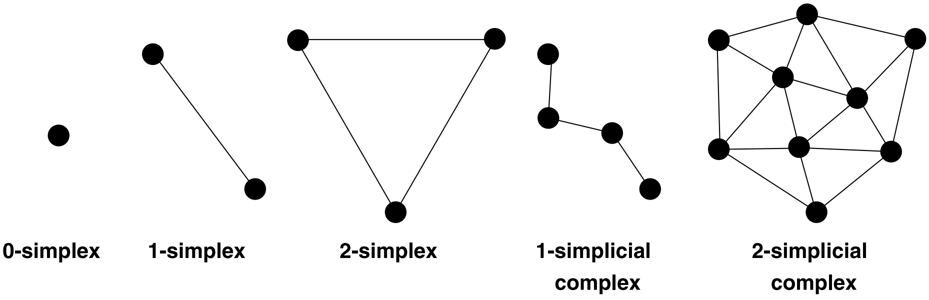

Fig. 1 Examples for simplexes and simplicial complexes

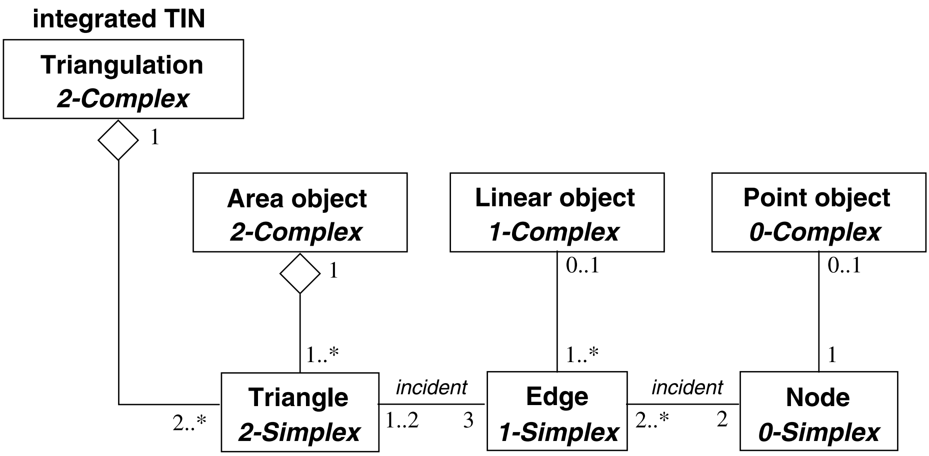

Fig. 2 2.5D-GIS-TIN data model

1.3 Structure of the Paper

In Section 2 the state-of-the-art in integrating 2D GIS data sets with height information is presented. Approaches relevant to our own investigations are described and evaluated in some detail. The section also contains a short description of data reduction algorithms which are important in our context due to the rather large data volume resulting from the integration. Section 3 is devoted to a new approach to integrate 2D GIS data and DTM called *radial topology algorithm*. After a description of the algorithmic background the advantages and limitations are discussed, and some enhancements are suggested. Section 4 describes various results from experiments with the radial topology algorithm as well as an associated discussion. Section 5 provides some aspects on which the proposed data model may be assessed, Section 6 concludes the article and gives an outlook for future research and development.

2 Related Work

2.1 Early Investigations

The basic idea of integrating DTM and 2D GIS data has already been mentioned by Peucker et al. [29] in their well known work on TIN. These authors felt that a combination of a TIN and a “polygon system” would incorporate the advantages of both concepts. Substantial development, however, only occurred at a later time.

Two integration approaches were discussed by Buziek [7]. The first one is based on the so-called *hybrid DTM* which is basically a gridded DTM enriched with structure lines (break lines, ridge lines etc.) to depict the relief in a morphologically sound manner. In the integrated model, DTM grid cells affected by 2D GIS geometry (here: the structure lines) are triangulated locally to represent the line information (e.g., [8],[14]). Grid cells and triangles are subsequently linked in some way to landscape objects and hence, height information may be analyzed object-based. Restricting this approach to solely triangles leads to the integrated model based on TIN which is the focus of this paper.

The other approach mentioned in Buziek [7] is the integration based on *polynomial surface objects*. This approach is basically an extension of height attributing to area objects, i.e., height step objects (see above). Polynomial surface objects are obtained by linking higher order polynomials to these objects, and an integration may again be conducted by a map overlay. The result is a better approximation of the shape of the relief, however, also polynomial surface objects of higher order show height discontinuities at the object borders comparable to the mentioned "wedding cake" approximation.

The two approaches were investigated in detail by Lenk [24]. The result indicated that the integration of 2D GIS data and TIN is superior to an integration based on polynomial surface objects, mainly because integrated 2.5D-GIS-TIN models the landscape surface without height discontinuities and require less computing effort.

2.2 Approaches to Integrate a DTM-TIN and 2D GIS Data

Pilouk [30] introduced the concept of modelling space by *simplicial complexes* (called *simplicial networks* in his work). The suggested procedure to establish an integrated 2.5D model first combines the nodes of the 2D GIS data set with the TIN nodes. Then, a *constrained triangulation* using the 2D line segments as constraints is performed. After interpolating a height from the original TIN for each node of the 2D GIS data set, the result is recorded as a 2.5D data structure.

Rather than combining all nodes before triangulating the data, Klötzer [21] (see also Plümer and Gröger [31]) introduce the nodes of the 2D GIS data set by incremental DT. Afterwards, the Delaunay-criterion is re-established, then the edges of 2D GIS data are added by splitting them with Steiner points at the intersection with the TIN edges. Steiner points are supplementary points which are inserted into existing triangulations.

As an important property of the integration process, Klötzer [21] stipulated that the shape of the DTM should not be altered during the integration process in order to preserve the terrain representation. However, the re-establishment of the Delaunay-criterion may lead to minor alterations of the original DTM-TIN shape. However, leaving out this step, the procedure fulfills the requirement of shape invariance. In the following discussion the latter procedure will be referred to as the *revised* approach of Klötzer [21].

The method of Egenhofer et al. [15], originally developed in another context, can be used for our task and is also incremental in nature, and is thus similar to the one of Klötzer. To embed the 2D data into the TIN, firstly the two nodes incident to an edge are inserted and afterwards Steiner points are computed at the intersection between these edges and the TIN edges. A complete polygon is inserted into the TIN by adding its edges sequentially.

Abdelguerfi et al. [1] conducted their work on the background that the *vector-product-format (VPF)* existing at the time did not meet the requirements of the modelling and simulation community of the US forces. They therefore introduced the so-called *extended-vector-format (EVPF)*. For each area object a local TIN is computed and the heights to be determined are interpolated linearly in the existing surface. "The process regions with a polygon with absolute x, y boundary coordinates. After determining which triangles contain these points, the polygon is overlaid by dividing it into one or more child polygons based on the elevation TIN edges. Once the child polygons are defined they are triangulated...” (Abdelguerfi et al., [1], p. 668). To triangulate the child polygons a procedure described in O’Rourke [28] is adapted.

2.3 Discussion of the Described Approaches

A closer look to the existing approaches reveals that generally they are capable of preserving the shape of the existing DTM-TIN, the only exception being the approach by Pilouk [30]. Inserting only the line segments of 2D geometry as constraints into the DTM-TIN may change the surface shape, if structure lines are also present, as one can see by the following example: Imagine a road crossing a ridge line. In the integrated model the ridge line will not exist anymore as the line representing the road takes priority. As a consequence, the algorithmic approach of Pilouk [30] needs correct input data in the sense, that if structure lines are present, intersection points between the 2D GIS data and these structure lines must be available prior to the integration. The revised procedure of Klötzer [21], the approach of Egenhofer et al. [15] as well as the method by Abdelguerfi et al. [1] do not suffer from problems of changing surface shape, however, they ignore certain special geometric constellations. These aspects will be treated in the following.

2.3.1 Special Geometric Constellations

When merging 2D GIS data and DTM-TIN, nodes from both data sets may lie at geometrically identical positions in 2D. Nodes of the 2D GIS data may also fall onto existing edges in the DTM-TIN and edges in both data sets may be completely or partially collinear. The insertion of nodes onto existing edges is a standard operation in computational geometry (e.g., de Berg et al., [2]) that is taken into account explicitly by Egenhofer et al. [15], and also identical nodes are considered by these authors. Klötzer [21] assumes that the input data sets are geometrically disjunct which is the common case in computational geometry when triangulating point sets. However, due to the independent capture of 2D GIS and DTM data it cannot be generally assumed that the nodes in both data sets are spatially disjunct. Edge collinearity is neglected in both approaches. With respect to these special geometric cases the procedure of Abdelguerfi et al. [1] heavily relies on the implementation of the map overlay. If the overlay algorithm does not consider these constellations faulty results may be obtained.

2.3.2 Algorithmic Aspects

Analyzing the algorithmic background of the procedures shows that they all neglect existing topological relations in both input data sets. For example, Klötzer [21] first inserts *all* nodes of the 2D data set into the TIN. He seemingly computes the location of insertion for *each* node without considering the inherent line topology in the 2D data set. Egenhofer et al. [15] first insert the start node of an edge and then its end node before computing intersection points with the TIN edges. The approach of Abdelguerfi et al. [1] is not described in detail, but it is algorithmically completely different as their approach relies on the map overlay process. To conduct a map overlay in an efficient manner, some sorting of input data has to be performed before computing the actual overlay itself. A possible sorting order for line segments is along one coordinate to compute the overlay afterwards with an output sensitive plane sweep algorithm (e.g., [2]). Hence, sorting all existing edges will commonly not use the existing topology of the input data sets.

Locating the triangle for insertion of the first node of a 2D line provides also an approximate location for the second node of that line. Hence, applying the strategy of *walking in a triangulation* that utilizes the TIN topology to navigate in the latter (e.g., [12]) should speed up the procedure substantially. The approach of starting a topological walk in a triangulation with a predetermined triangle is also called the *jump-and-walk* strategy (e.g., [23]). Hence, implementing an algorithm that traverses all geometries of a 2D GIS data set and steps along the nodes of the geometries while walking simultaneously in the TIN exploits the topologies of both input data sets advantageously. Additionally, no special pre-sorting of geometric primitives has to be conducted.

2.4 Data Reduction of Input Data

As mentioned before integrated data sets tend to have large data volumes. One way to minimize data volumes is to reduce the amount of input data separately, i.e., to simplify the 2D GIS data and the DTM data before integrating them.

One well known procedure for 2D line simplification is the algorithm developed by Ramer [33] and Douglas and Peucker [13], others are summarized e.g., by [35]. The Ramer- or Douglas–Peuker-algorithm is well-known, thus we do not describe it here, although we will make use of it later (see Section 4).

Reduction of DTM data is particularly important, if a gridded DTM is available as input, as gridded DTM often contain a large amount of redundancy. Adaptive triangulations used for data reduction in DTM-TIN are treated by [1],[20],[32] and others. We use the approach developed by Lenk and Kruse [25]. Comparable to the scheme of the Ramer- or Douglas–Peucker-algorithm a point set in the plane is initially approximated by its convex hull (CH) which may be illustrated by a rubber band containing all input data. For a gridded DTM the CH is given by its corners. The CH is triangulated, resulting in an initial approximation of the surface described by the input point set. For the remaining points the vertical distance (rather than the perpendicular distance which is better for highly undulated terrain, but computationally more involved) to the initial surface is computed. Similar to the [13] algorithm the point with the maximum distance is inserted, and the remaining distances are re-computed. The algorithm is repeated, until the maximum distance is below a pre-defined threshold.

3 A New Approach to Compute the Integrated Model

From the above considerations a new method to integrate existing piecewise linear 2D geometries into a TIN has been developed (see also [24]). It takes into account special geometric constellations, fully exploits existing topologies of input data and works similar to the *jump-and-walk* strategy. The algorithm "sews the 2D data into the TIN like a sewing-machine" while traversing the latter along the 2D data.

In Section 3.1 we describe the new method based on a single triangle, Section 3.2 explains how the method works with a number of successive nodes of a 2D geometry and demonstrates that the result contains a certain amount of redundant nodes. After presenting the data model with the minimum number of nodes (Section 3.3) an enhancement of the new method which leads to non-redundant results is presented in Section 3.4.

3.1 Description of the Algorithm

The basic principle of the algorithm is illustrated by Fig. 3: The area of a triangle and its adjacent neighbors as well as its incident, oriented edges E_0, E_1 and E_2 and nodes P_0, P_1 and P_2 may be subdivided into 19 distinct geometric locations. The basic primitive for this operation is the determinant computed by an oriented edge of the triangle and the node to be tested [28]. The sign of the determinant provides information whether the node lies to the left or right of the respective edge and in addition, it delivers the area of the triangle given by the edge and the node. Therefore, if the area is equal to zero, the tested node is collinear to the edge. Combining all the three determinants computed from the node to be tested and the three triangle edges provides information in which of the 19 locations the node lies. If the node lies outside the center triangle, the combination of determinants delivers the adjacent triangle which serves as input for the next determinant test. Using this strategy, one can navigate in a TIN to a destination given by the coordinates of the 2D geometry.

Modifying this approach leads to a procedure that integrates 2D data into a TIN. Figure 3 may be re-drawn in the way given in Fig. 4.

By knowing an incident triangle the horizon around a node may be divided into distinct sectors represented by its incident edges and other geometric locations. The basic primitive for this operation again is the signed determinant. The respective determinants computed by the edges of the incident triangles and the end node of the current line segment to be integrated into the TIN provides information where the next node is located:

1. In the triangle itself (case 1), so the node is simply added to the TIN and the procedure continues with the next node of the 2D geometry;
2. On an incident edge of the last node (case 2a/b), hence the current line segment is partially collinear with existing edges;
3. On adjacent nodes of the last node (case 3a/b), i.e., the next node of the current segment is identical to an existing node in the TIN;
4. On the extension of the incident edges of the node (case 4a/b), i.e., the current line segment needs to be split by the adjacent node as the incident edge is partially collinear to the line segment;
5. In the sector of an adjacent triangle (case 5a/b), the search will continue with a triangle whose index is determined by use of the TIN topology;
6. On the edge opposite the node (case 6);
7. Beyond the edge opposite the node (case 7); consequently, the current line segment is split by the opposite edge and an intersection point must be computed;

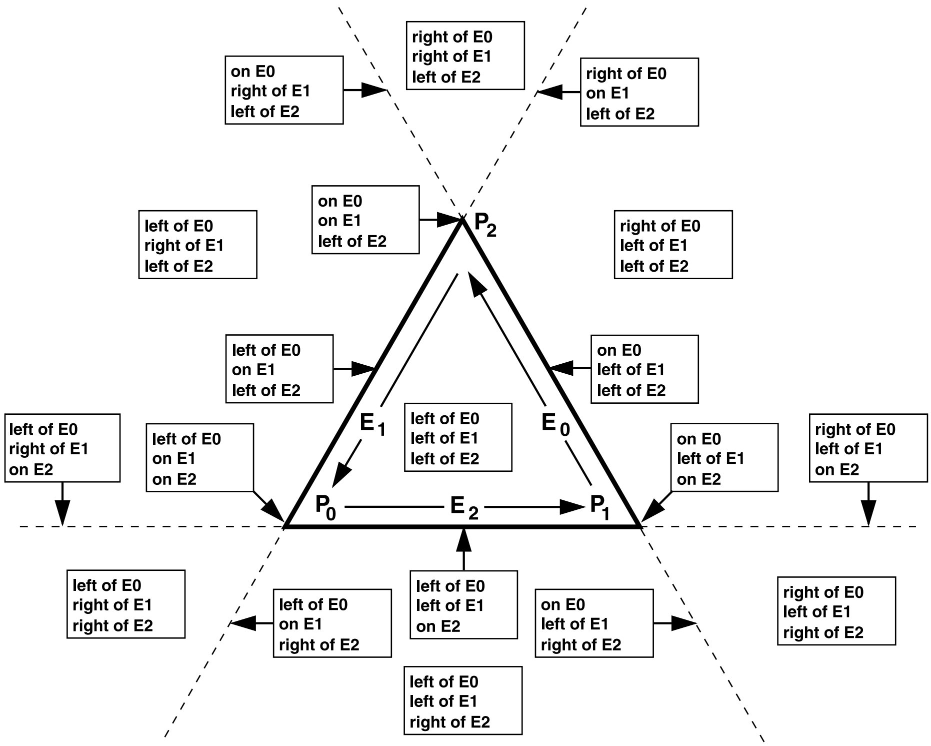

Fig. 3 Dividing the horizontal plane around a triangle into decision areas

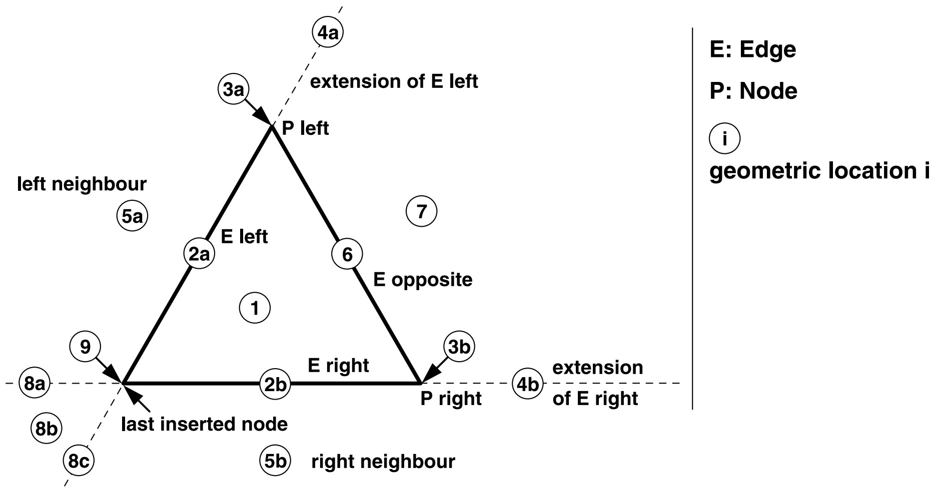

Fig. 4 Radial-topological search around a node

8. To the back of the current node (case 8a/b/c), the search continues with an adjacent triangle, similar to case 5;
9. On the current node, i.e., both nodes are geometrically identical (case 9).

On the basis of these events all geometries may be integrated into the mesh by traversing the object holding the 2D geometries. As the basic operation in this algorithm is the radial sweep combined with a topological walk along the 2D geometry and in the TIN, the algorithm is termed *radial topology algorithm*.

In contrast to other approaches, the described procedure considers the partial collinearity of edges of both input data sets explicitly as respective edges are split by existing nodes. A further advantage is the fact that in its geometric part the influenced area of a single insertion has at maximum the extent of four triangles (in case the respective node lies on an edge), which leads to small I/O traffic.

A prerequisite of the radial topology algorithm is that the complete 2D data must be situated inside the convex hull of the DTM-TIN. Also, the algorithm only inserts 2D geometries (instead of landscape objects) into an existing DTM-TIN. To derive a fully object-based landscape model, the different simplexes still need to be linked to their respective objects and vice versa. Whereas point features may easily be linked to nodes in the integrated model during insertion, and linear features can be linked with moderate effort to 1-simplexes while traversing the DTM-TIN, the situation becomes more involved when area or surface features have to be assembled from the triangles in the integrated models.

Generally there are two possibilities to establish links between area objects and triangles. The first way is conducted on the basis of *point-in-polygon-tests*. For each triangle its center is computed by averaging the coordinates of its corner points, and the point-in-polygon-test is then carried out to determine the object in which the triangle center lies [30]. The other possibility utilizes the topology of the resulting integrated TIN. For each object, a so-called virus (cf. [34], 1997) is set up at an object boundary. It then searches recursively for triangles until it reaches another boundary to neighboring object.

3.2 Geometric Analysis

The explanations in Section 3.1 dealt solely with an individual triangle incident to the current node being processed. It is now the aim to demonstrate how the algorithm works with a number of successive nodes of a 2D geometry in a triangle of the DTM-TIN. A possible situation is given in Fig. 5.

Nodes no. 2 and 4 are nodes of a 2D geometry entering the triangle from North-East fall into the triangle. In the integrated model the 2D geometry is represented by nodes no. 1, 2, 3, 4, 5 and 6, the numbering of the nodes indicates the sequence of insertion (the sequence may be evaluated by use of Fig. 4). Nodes no. 1 and 6 are intersection points with existing edges of the original DTM-TIN and are hence necessary for the tesselation of the horizontal plane into area objects. Nodes no. 3 and 5 do not contribute additional information to the tesselation of the plane, as they lie on edges of the 2D geometry, and they do not add information to the description of the surface shape of the original DTM-TIN as their heights were interpolated linearly in the original DTM-TIN-triangle.

Thus, the radial topology algorithm leads to redundant data if intersection nodes are computed between the geometry to be added to the DTM-TIN and edges that

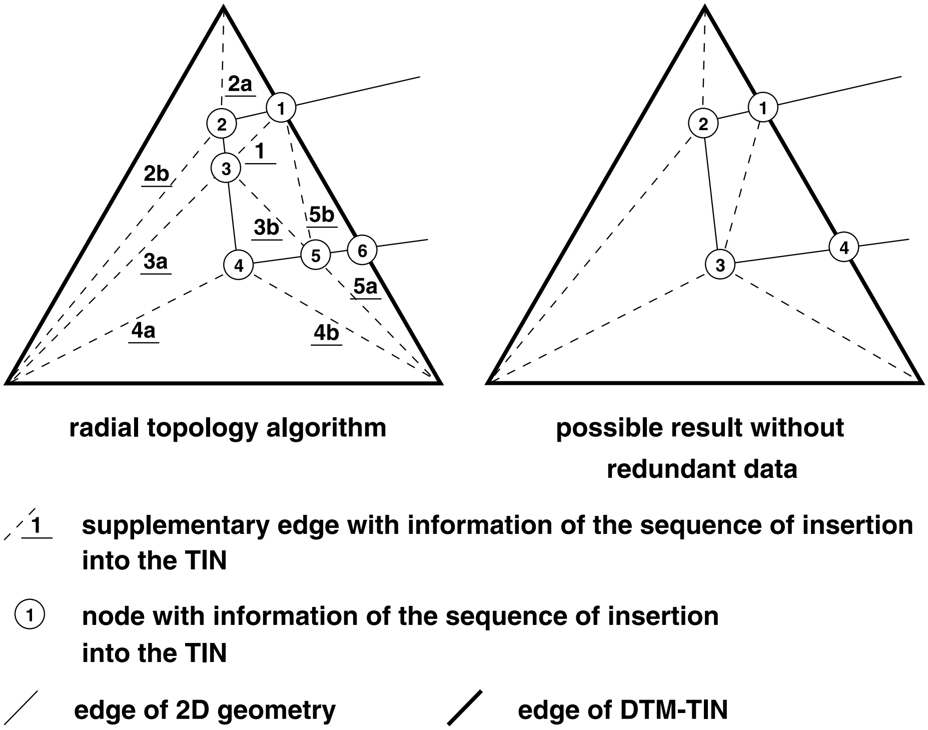

Fig. 5 Geometric analysis of the results of the radial topology algorithm

have been added to the integrated TIN beforehand. Examples of integrated models in Lenk[24] showed that the amount of redundant nodes may be more than half of the total amount. As a consequence, it is highly recommended either to avoid or to delete redundant data, in particular with respect to the set up of national core geospatial databases and their already large data volume.

It should be noted that also the approaches of Klötzer [21] and Egenhofer et al. [15] lead to redundant data. Whereas the data volume of Egenhofer et al. [15] is the same as for the radial topology algorithm, the revised approach of Klötzer [21] creates a data volume differing from the two others as first *all* nodes of the 2D data are inserted into the TIN. The resulting data volume is in general larger because a maximum of intersection candidates is introduced into the TIN at each state of algorithm execution. Only the procedure of Abdelguerfi et al. [1] does not produce redundant data.

3.3 The Data Model with the Minimum Number of Nodes

From the section above it can be concluded that Steiner points added during the integration are valid if and only if they are intersection points between edges of the 2D GIS data and DTM-TIN edges. This relation has been considered in the integrated data model which is presented in Fig. 6 using the UML.

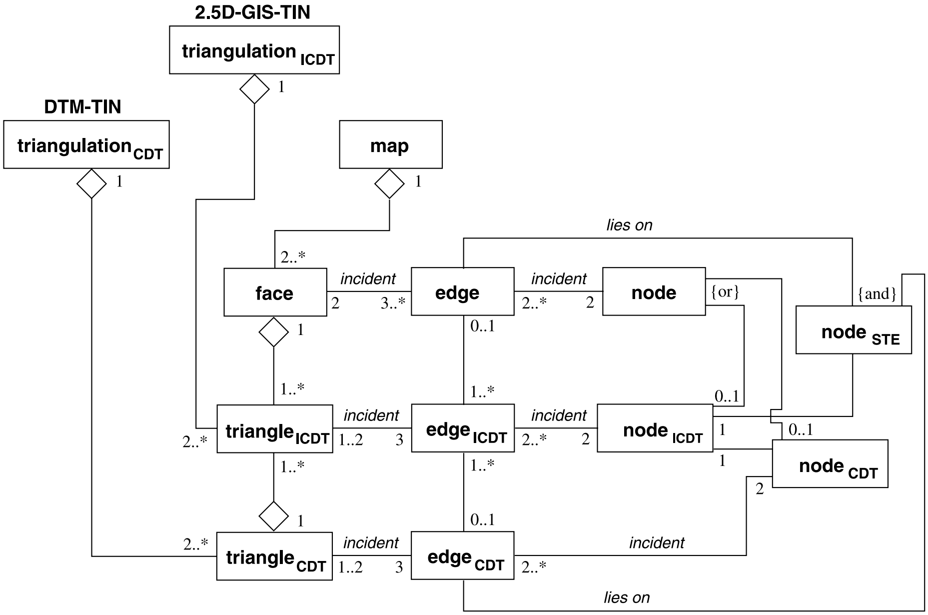

Fig. 6 Integrated data model with minimum number of nodes

The DTM-TIN is represented by a CDT to incorporate structure lines into the DTM-TIN. The latter and the 2D GIS data must be treated as independent input data sets in the resulting integrated model, the 2.5D-GIS-TIN. Since the DELAUNAY-criterion is not re-established during the integration, the resulting triangulation is constrained even if the DTM-TIN does not contain morphological information, hence the computation is termed the integrated constrained DELAUNAY-triangulation (ICDT). A node of the 2D GIS data may fall onto a DTM-TIN edge or on a node of the TIN. Hence, edges of the 2D data may be partially collinear with DTM-TIN edges. A so-called "map" [31] is decomposed into one or more faces, i.e., polygons, and each face consists of one or more triangles. The relation that only intersection points are valid STEINER points is expressed by the and-relation between the incident edges of the STEINER points $\mathbf{N}ode_{STE}$.

3.4 How to Compute the Integrated Model with Minimum Number of Nodes

In this section we describe ways to extend the radial topology algorithm in order to avoid the creation of redundant data. In principal there are two ways to proceed: one can compute the non-redundant result (1) during insertion of the 2D geometries into the DTM-TIN, or (2) in a separate step based on the intermediate, redundant result.

In order to avoid redundant data during insertion, it is necessary to insert the first nodes of the geometry temporarily until the geometry leaves the starting DTM-TIN triangle. At this stage it must be checked whether redundant data was generated inside the triangle. If this is the case, re-triangulation has to be conducted while suppressing the redundant data. This approach is mandatory if a simple triangle-based data structure is used that stores only topological relations between triangles and pointers from triangles to nodes, as we have done. It can be shown that the approach is algorithmically rather involved, in particular for linear objects.

Therefore, we have instead developed a procedure for the second alternative as indicated in Fig. 7. While being significantly less complex in algorithmic terms, the disadvantage of this approach is the additional computational effort, resulting from the fact that we need to walk along the 2D data twice.

Redundant nodes are marked as such during the initial step of integration. To delete them from the intermediate integrated model the procedure sweeps radially

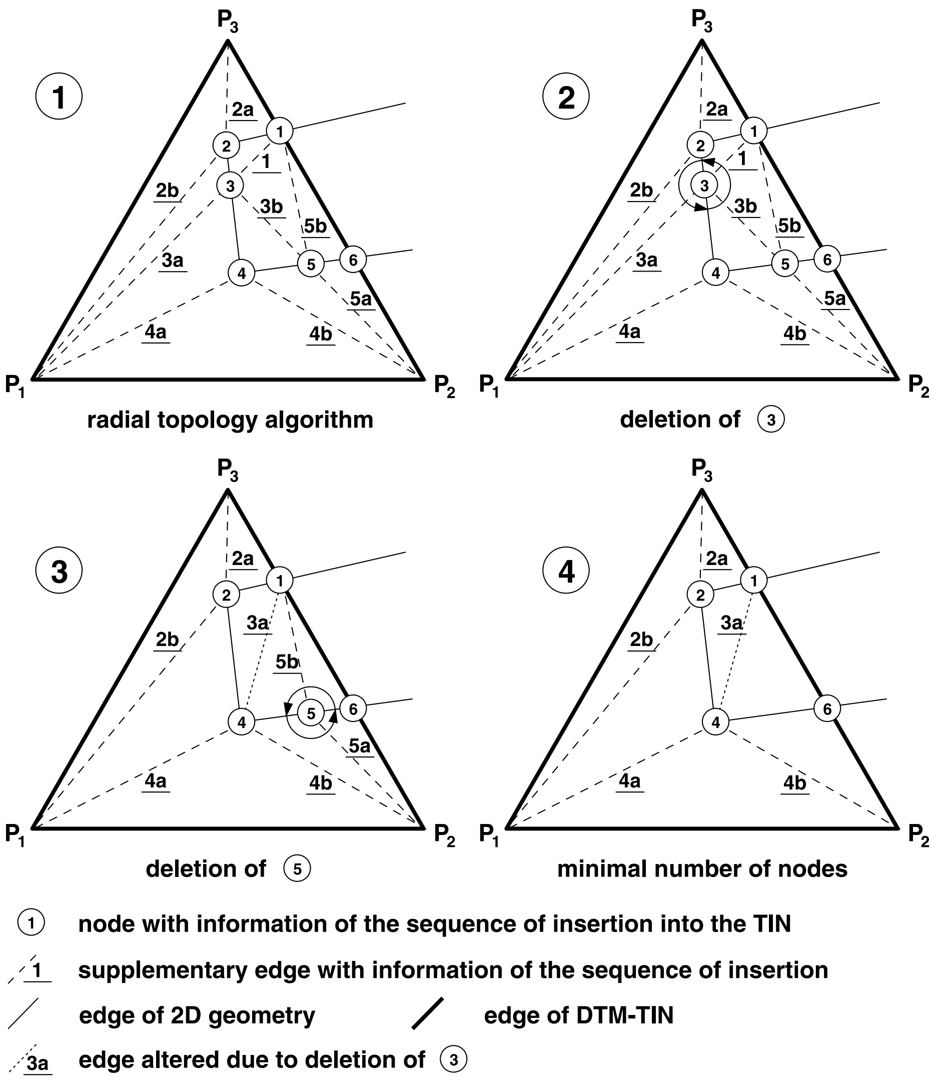

Fig. 7 Deletion of redundant data from Fig. 5

around each node to collect incident and adjacent primitives, and then perform a polygon triangulation. During the polygon triangulation this step is necessary to consider the special case of three successive collinear nodes in the polygon. On the basis of the mentioned signed determinants such situations can be found and treated in an appropriate way. As a result a non-redundant integrated 2.5D-GIS-TIN is available.

4 Experimental Results

In this section we show results of the integration. We also come back to the issue of data reduction as indicated in Section 2.4. This has proven to be of particular relevance, because we work with gridded DTM as input.

Figure 8 shows the input data sets as well as two integrated models for one of our test areas, the Leine floodplain, south of Hannover, Germany. The Leine running south along the base of a small mountain can be easily identified. The eastern part of the area shows the floodplain with low relief energy. The upper left part of Fig. 8 shows the 2D GIS data, in the upper right the resulting integrated model is depicted. The underlying DTM has of course been triangulated, but the gridded nature of the input is still clearly visible. It also becomes clear that from a topographic point of view this result contains a high amount of redundant data (note, that this redundancy is due to the grid data model and has no connection to the algorithmic redundancy discussed in the previous section).

Applying data reduction techniques as described in Section 2.4, namely the Ramer- / Douglas-Peuker algorithm for the 2D GIS data, and adaptive triangulation for the DTM-TIN (see again [25]), has yielded the two lower representations of Fig. 8. In the lower left the DTM-TIN, adaptively triangulated with a threshold of 1 *m*, can be seen, in the lower right, the integrated model resulting from the simplified 2D data set (threshold 1 *m*) and the adaptively triangulated DTM-TIN is depicted. It is clearly visible that the data volume of the lower right model is significantly smaller than the one depicted in the upper right, as was to be expected. A feature of TIN-based integrated models can also be observed: especially in flat areas the approach can lead to long skinny triangles. This is due to the different point densities along the 2D geometries and the spatial distribution of DTM-TIN nodes in the horizontal plane.

A deeper insight into numerical results with respect to data volume is given in Tables 1 and 2 which contain not only information for the Leine area but also for the other test areas Damme, situated in a hilly moraine landscape, and Ebergötzen representing a mountainous region in the Harz mountains, both in Germany. Table 1 contains information about the input data. All three test areas are of equal size. As the number of nodes in a grid is independent of the morphology, i.e., it depends only on its size, it is only provided for the first area.

It should be noted that the height accuracy of DTM data used as input was stated with 1 *m* by the data provider, the respective value for horizontal accuracy of 2D data is 3 *m*. Therefore, applying a threshold of 1 *m* for simplification of both data sets should not deteriorate the general accuracy of the integrated model. The second step of simplification (3 *m*) was computed to show the result according to the specified accuracy of the 2D input data set.

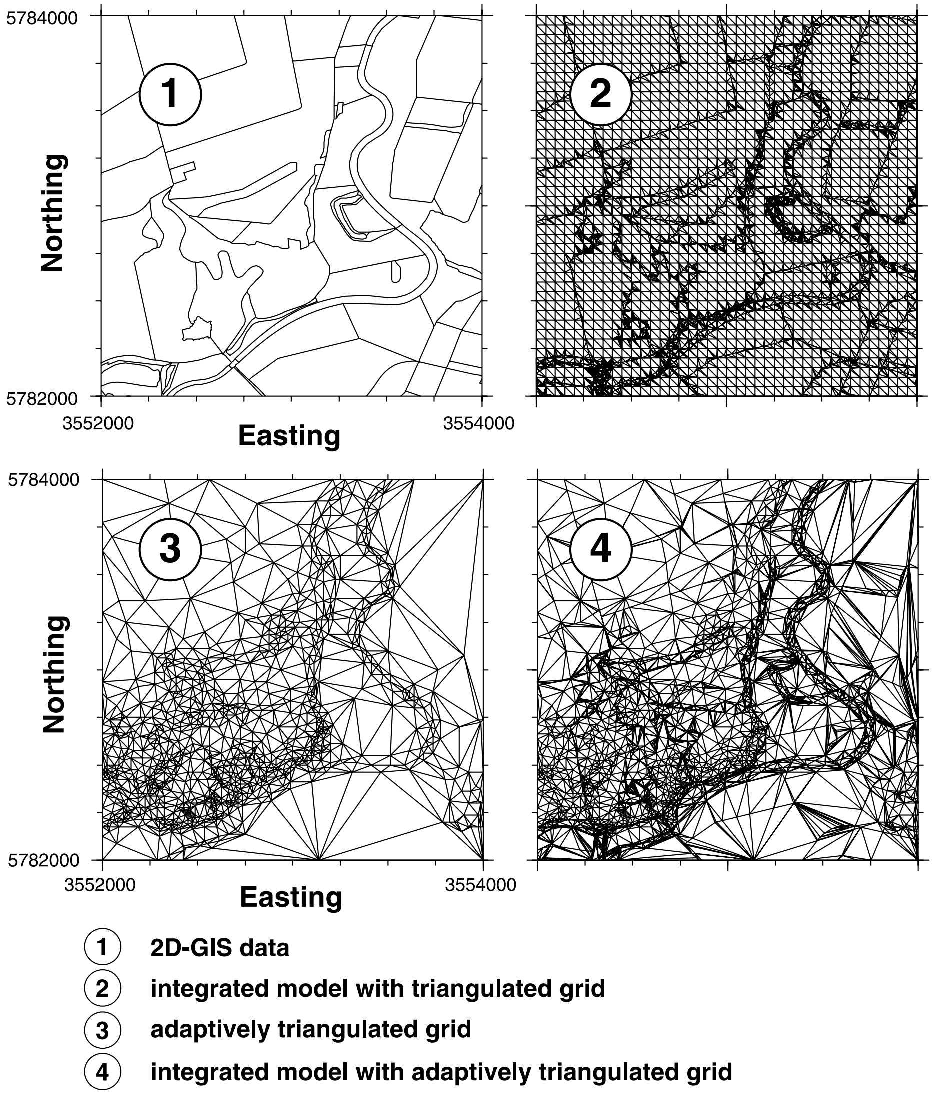

Fig. 8 Examples of integrated models for test area Leine

In Table 2 the size of the resulting integrated models is shown. Taking into account the different morphology and density of the 2D GIS data, comparable results are obtained for all three test areas. It is clearly visible that applying data reduction algorithms to input data sets leads to a significantly smaller amount of nodes in the integrated models and that adaptive triangulation contribute the major part in data simplification. Besides, the effect of data reduction of course depends on the terrain morphology. Areas with a higher relief energy lead to a denser node distribution in the adaptive TIN and thus in the respective integrated models.

Table 1 Amount of nodes in (simplified) input data sets for different test area

Test area	Type of data	Threshold [*m*]	Nodes abs.	Nodes [%] of full data set
Leine/Damme/Ebergötzen	full 12.5-*m*-grid	0	25,921	100
Leine	simplified DTM-TIN	1	1,103	4.3
Leine	simplified DTM-TIN	3	265	1.0
Leine	full 2D data set	0	1,608	100
Leine	simplified 2D data set	1	947	58.9
Leine	simplified 2D data set	3	620	38.6
Damme	simplified DTM-TIN	1	2,301	8.9
Damme	simplified DTM-TIN	3	587	2.7
Damme	full 2D data set	0	2,323	100
Damme	simplified 2D data set	1	1,230	52.9
Damme	simplified 2D data set	3	841	36.2
Ebergötzen	simplified DTM-TIN	1	4,109	15.9
Ebergötzen	simplified DTM-TIN	3	1,337	5.2
Ebergötzen	full 2D data set	0	2,962	100
Ebergötzen	simplified 2D data set	1	1,587	53.6
Ebergötzen	simplified 2D data set	3	1,020	34.4

5 General Discussion of Integrated Models Based on TIN

The aim of this section is to discuss the compatibility of the TIN-based integration of DTM and 2D GIS data with existing concepts in GIS research and practice. Rather than developing a general catalogue of features to assess the data model, a small and not necessarily complete list of properties is used. For the decision in a specific project, these elaborations can be useful in an initial assessment phase, they should however be extended in light of any particular requirements.

Compatibility with existing data sets is provided as it was one of the major prerequisites for developing the 2.5D geo-spatial data model. The model uses input data

Table 2 Amount of nodes in integrated data models (with simplified input data sets)

Test area	Type of input data	Threshold [*m*]	Nodes abs.	Nodes [%] of full data set
Leine	12.5-*m*-grid as TIN, full 2D data	0	34,166	100
Leine	simplified DTM-TIN and 2D data	1	3,143	9.2
Leine	simplified DTM-TIN and 2D data	3	1,314	3.8
Damme	12.5-*m*-grid as TIN, full 2D data	0	36,656	100
Damme	simplified DTM-TIN and 2D data	1	5,513	15.0
Damme	simplified DTM-TIN and 2D data	3	2,285	6.2
Ebergötzen	12.5-*m*-grid as TIN, full 2D data	0	37,661	100
Ebergötzen	simplified DTM-TIN and 2D data	1	8,555	22.7
Ebergötzen	simplified DTM-TIN and 2D data	3	3,918	10.4

as provided by national mapping agencies, and it may be easily applied to other data sources such as irregularly sampled DTM data.

Compatibility with existing geo-spatial data models and GIS software is provided under certain restrictions. Simplicial complexes, and thus TIN, may be stored in geo-databases if the latter are capable of processing complex objects which consist of several geometric parts. Otherwise, they must be stored as separate TIN where each TIN triangle is linked with the full attribute data set of the respective parent object.

Compatibility with existing approaches in geo-spatial data modelling: 2-simplicial complexes are compatible with higher dimensional data models as the theoretic background may be extended to 3D and 4D, see [30], and [5]. 2-simplicial complexes can also be used to model general surface features in space where a multitude of height values is allowed for each position in the horizontal plane (see [27]), however topology is still purely two-dimensional. Such approaches are sometimes termed as *2.75D* (e.g., [9],[18]). Hence, the proposed procedure of integration on the basis of triangulations is also compatible with existing approaches in higher dimensional GIS research, and also provides for migrating existing national core geo-spatial databases to 3D.

Quality of terrain modelling using TINs is influenced mainly by three factors: firstly by quality of the original input data, secondly by effects caused by the interpolation scheme being used to interpolate the surface between original data points, and possibly thirdly by a data simplification scheme if it is applied. This is also the case for 2D data whereas not a surface but a smooth line is being interpolated between original points. In case data simplification is required, the use of similar threshold values for both input data sets allows to construct models with homogeneous accuracy for all three dimensions. As a consequence, quality is easily controlled in TIN-based 2.5D landscape modelling.

The computational effort to compute TIN-based integrated models is governed by the way the DT is computed and the spatial access data structure being used for the incremental insertion of nodes. As the DT of irregular point sets and triangulation of polygons is a well-researched field in computational geometry, many working algorithms are available. Algorithms for DT may be implemented as $O(n \log n)$ processes with moderate effort, and the insertion of additional nodes has only $O(n^2)$ in the worst case (e.g., [19]).

Analysis and spatial query space: As the TIN is a well-researched data model, many algorithms exist to analyze this data structure (volumes, isolines, surface contents etc.). 2.5D-GIS-TIN may be easily accessed spatially as they describe objects with discrete nodes. Objects are represented by boundary representation or by sequences of nodes.

Data volume: The integration based on triangulation leads sometimes to a large data volume, as was discussed in the previous section. We have also shown that the data volume may be reduced algorithmically and controlled efficiently by appropriately simplifying the input data.

Visualization of TIN is a standard procedure and consequently, no technical problems are anticipated in this area. However, care should be taken in larger scales, where semantic problems may appear, if the 2D GIS data and the DTM are not consistent with each other and with the conception of a human observer. For instance, the height information taken from the DTM for a 2D polygon denoted as a lake is not necessarily flat, and this semantic inconsistency may lead to artifacts in the visualization.

As a result of the discussion in this section, we can state that the integration based on triangulations fits into existing approaches of capturing geo-spatial information, higher dimensional data modelling and management as well as analysis and visualization. By using integrated data potential users will benefit from an improved access to the third dimension based on individual landscape objects.

6 Conclusions and Outlook

In this article we have described a new algorithm, termed the *radial topology algorithm*, for the generation of an integrated 2.5-GIS-TIN data set starting from separate DTM and 2D GIS data. Besides the application of integrating 2D data into a TIN, the algorithm is capable of inserting any 2D geometry into existing triangulations, and to interpolate height information efficiently for successive nodes in a 2D topological network such as a road network used in car navigation. Except for the differences in height interpolation, it can also be used to integrate structure lines into TIN.

We have discussed the advantages and limitations of the radial topology algorithm, also with respect to approaches in the literature, and have described ways to eliminate the redundant nodes generated during the integration process. With the help of four examples from practical work we have shown that it is feasible to compute and work with such integrated data sets.

We have also discussed the integrated data models in the light of various more general requirements. As a consequence of this discussion, we conclude that the integration based on triangulations fits into existing approaches of capturing geo-spatial information, higher dimensional data modelling and management as well as analysis and visualization. By using integrated data potential users will benefit from an improved access to the third dimension based on individual landscape objects.

The next steps of our work comprise mainly two points: we hope that many different DTM and GIS users will benefit from our work. We want to make sure that these advantages can be brought to light more clearly with the help of prototype applications. Thus, we will seek collaboration with users in order to test and refine the described algorithm and data model.

The second issue is more research oriented. As was mentioned in the previous section, besides the geometric issues of integration there are also semantic questions, which need to be addressed. The example of the lake was already given above. Consider as another one a road running across a mountainous area. The 2.5D road should have a correct slope along and across its axis everywhere. Most probably this will not be the case, when the height of the corresponding nodes is taken from the integrated data model, since it must be assumed that the DTM and the 2D GIS are of different geometric accuracy and resolution, have been captured at different times, from different data sources, and for different purposes, to give just a few reasons for the resulting inconsistency.

The underlying problem is that although no explicit height information is stored in the 2D GIS data, the nature of the object class implicitly introduces height information: a lake is flat, as is a soccer field, a road can only have limited slopes and slope changes, rivers monotonously run downhill, overpasses need to actually pass above underpasses etc. Thus, the task is to formalize these implicitly given height cues and to introduce them into the integration process. Initial work on this topic has been reported by Koch and Heipke [22].

Yet another issue worth mentioning is the extension of the model to higher dimensions (3D and 4D) using other or additional data sources as well as application oriented models (e.g., a specific surface model for car navigation similar to the approach of Chen et al. [10]). Thus, the topic of integrating planimetric and height information including their temporal behavior to finally arrive at true 3D or 4D representations of our world remains a complex and very interesting challenge.

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